4. Probability distributions of continuous random variables

Uniform distribution

$$f_X(x) = \frac{1}{b-a}, \quad F(x) = \frac{x-a}{b-a}, \quad x \in [a,b] \subset \mathbb{R}$$

Properties: $m_X = \frac{a+b}{2}$, $\mathrm{Var}(X) = \frac{(b-a)^2}{12}$.

Exponential distribution

$$f_X(x) = \theta \mathrm{e}^{-\theta x}, \quad F(x) = 1 - \mathrm{e}^{-\theta x}, \quad x \geq 0$$

Properties: $m_X = 1/\theta$, $\mathrm{Var}(X) = 1/\theta^2$.

Normal (Gaussian) distribution

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\frac{(x-m)^2}{2\sigma^2}}, \quad F(x) = 0.5 + \Phi\left(\frac{x-m}{\sigma}\right) = 0.5 + \Phi(z), \quad x \in \mathbb{R}$$

Properties: $m_X = m$, $\mathrm{Var}(X) = \sigma^2$.

Standard normal random variable: $Z = (X-m)/\sigma$.

Standardizing the normal distribution: $\mathcal{N}(X; m, \sigma) \to \mathcal{N}(Z; 0,1)$.

Laplace function (tabulated): $\Phi\left(\frac{x-m}{\sigma}\right) = \Phi(z) = \frac{1}{\sqrt{2\pi}} \int_0^z \mathrm{e}^{-\frac{u^2}{2}} \mathrm{d}u$; $\Phi(\infty) = 0.5, \Phi(-z) = -\Phi(z)$.

Approximations by normal distribution

Binomial distribution can be approximated by normal:

- if the probability of success p is close to 0.5 and the number of trials n is large,
- if p is not close to 0,5, but the following is true: $np > 10$ and $n(1-p) > 10$.

In that case, the normal distribution parameter values are: $m = np$ and $\sigma = \sqrt{np(1-p)}$.

Poisson distribution can be approximated by normal, if $\lambda > 5$.

In that case, the normal distribution parameter values are: $m = \lambda$ and $\sigma = \sqrt{\lambda}$.

4. Probability distributions of continuous random variables – problems

Note: To solve some of the problems, a tabulated Gaussian probability distribution is required. It is published on the Random phenomena web page.

1. The flange thickness on an aircraft component is uniformly distributed between 0.95 and 1.05 mm.
   a. Determine the cumulative distribution function of flange thickness.
   R: $F(x) = 10\ \mathrm{mm}^{-1}(x - 0.95\ \mathrm{mm})$
   b. Determine the proportion of flanges that exceed 1.02 mm. R: $P = 0.3$
   c. What thickness is exceeded by 90% of the flanges? R: $x = 0.96\ \mathrm{mm}$

2. Time to failure of a computer hard disk is exponentially distributed with a mean of 25 000 h.
   a. What is the probability that the disk runs without failure for at least 30 000 h? R: $P = 0.301$
   b. What is the time to failure that at most 10 % of disks exceed? R: $t = 57565\ \mathrm{h}$

3. Two weeks after being sowed, the mean plant height is 10 cm with the standard deviation of 1 cm. It is assumed that the plant height is normally distributed.
   a. What is the probability that the height of a randomly chosen plant falls between 9 and 12 cm? R: $P = 0.819$
   b. What height is exceeded by 90 % of the plants? R: $h = 8.72\ \mathrm{cm}$

4. The probability of getting a bad product in a series of 1000 pieces is 0.02.
   a. What is the probability that more than 30 bad products are found in a randomly selected series? R: $P = 0.0126$ (exact) and $P = 0.0119$ (approx.)
   b. What is the minimum capacity of a warehouse in which all bad products from a selected series can be stored with a probability of 0.95? R: $C = 28$