Formulae used for tutorials in Random phenomena course – 1st set

Events and probability

Events

Nomenclature: S .. sample space/certain event; $\emptyset$ .. impossible event; A, B, C .. events ($A, B, C \subset S$).

Property	Intersection $A \cap B$	Union $A \cup B$
Commutativity	$A \cap B = B \cap A$	$A \cup B = B \cup A$
Associativity	$(A \cap B) \cap C = A \cap (B \cap C)$	$(A \cup B) \cup C = A \cup (B \cup C)$
Subset	$(A \cap B) \subset A \ \wedge\ (A \cap B) \subset B$	$A \subset (A \cup B)\ \wedge\ B \subset (A \cup B)$
Distributivity	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Misc.	$A \subset B \Rightarrow A \cap B = A$	$A \subset B \Rightarrow A \cup B = B$

Complement A^{C}: $\left(A^{\mathrm{C}}\right)^{\mathrm{C}} = A$, $A \subset B \Rightarrow B^{\mathrm{C}} \subset A^{\mathrm{C}}$,

$A \cap A^{\mathrm{C}} = \emptyset$, $A \cup A^{\mathrm{C}} = S$.

DeMorgan's laws: $(A \cup B)^{\mathrm{C}} = A^{\mathrm{C}} \cap B^{\mathrm{C}}$, $(A \cap B)^{\mathrm{C}} = A^{\mathrm{C}} \cup B^{\mathrm{C}}$.

Interesting: $(\cup, \cap, \subset, \supset, S, \emptyset) \overset{\mathrm{C}}{\Leftrightarrow} (\cap, \cup, \supset, \subset, \emptyset, S)$, e.g. $(A \cap B = \emptyset)^{\mathrm{C}} \Leftrightarrow A^{\mathrm{C}} \cup B^{\mathrm{C}} = S$.

Probability

Definition: $P(A) = \lim_{n\to\infty} p(A)_n = \lim_{n\to\infty} \frac{n_A}{n}$.

Properties: $P(A) \in [0,1]$, $P(S) = 1$, $P(\emptyset) = 0$, $P\left(A^{\mathrm{C}}\right) = 1 - P(A)$.

Probability of union: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Probability of intersection of **mutually exclusive** events: $A \cap B = \emptyset \Rightarrow P(A \cap B) = 0$.

Conditional probability of event A given the event B took place:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad \text{where} \quad P(B) > 0.$$

- Commutativity of intersection: $P(A \cap B) = P(A|B)P(B) = P(B \cap A) = P(B|A)P(A)$.
- A and B **independent** $\Leftrightarrow P(A|B) = P(A) \Leftrightarrow P(B|A) = P(B) \Leftrightarrow P(A \cap B) = P(A)P(B)$.

Mutually exclusive and exhaustive set of events $\{A_1, A_2, \cdots, A_n\}$:

$$\bigcup_{i=1}^{n} A_i = S \quad \text{and} \quad A_i \cap A_j = \emptyset \ \text{ for } \ i \neq j.$$

Bayes theorem in which $P(B)$ is expressed by mutually exclusive and exhaustive set $\{A_i\}$:

$$P\left(A_j|B\right) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(A_j)P\left(B|A_j\right)}{\sum_{i=1}^{n} P(A_i)P(B|A_i)}.$$

Problems for tutorials in Random phenomena course – 1st set

Events and probability

1. A worker is responsible for two machines at the same time. The probability that he needs to intervene in a given hour at the machine A is 0.7, while the probability of intervention at the machine B is 0.5. The probability that he has to intervene in a given hour at one or at both machines is 0.8.

    a. What is the probability that the worker in a given hour intervenes at the machine B but not at the machine A? R: $P = 0.1$

    b. What is the probability that the worker's intervention in a given hour is not needed? R: $P = 0.2$

2. On each of the 300 pieces the depth of bore is measured and its surface is described. The results are in the table.

		Bore		
		Too Shallow	Appropriate	Too deep
Surface	Rough	15	20	10
	Smooth	25	30	20
	Very smooth	60	50	70

    a. What is the probability that a randomly selected piece has a smooth surface and an appropriate bore? R: $P = 1/10$

    b. What is the probability that a randomly selected piece does not have a rough surface and/or does not have a too deep bore? R: $P = 29/30$

    c. What is the probability that among the pieces with a smooth surface a piece with a too shallow bore is randomly selected? R: $P = 1/3$

3. In a garage, cars are repaired by three workers. Each car is repaired by only one worker. Worker A repairs 35% of the incoming cars, worker B 40% and worker C the rest. Customers complain for 2% of the cars repaired by the worker A, for 4% of the cars repaired by the worker B and for 3% of the cars repaired by the worker C. We randomly select a repaired car.

    a. What is the probability that a complaint will occur for the selected car? R: $P = 0.031$

    b. If a complaint occurred for the selected car, what is the probability that it was repaired by the worker B? R: $P = 0.525$

4. Products are bought from manufacturers A, B and C. 40% of the products are bought from the manufacturer A, 30% from manufacturer B and the rest from manufacturer C. The products of manufacturer A are defective in 1% of cases, of manufacturer B in 4% of cases and of manufacturer C in 2% of cases. A product is randomly selected.

    a. If the selected product is defective, what is the probability that it has been manufactured by the manufacturer A or manufacturer C? R: $P = 0.455$

b. If the selected product is free of defects, what is the probability that it has been manufactured by the manufacturer B? R: $P = 0.294$

5. A machine consists of three components which operate independently. The probability that a component breaks down in a given day is 0.3 for the first component, 0.4 for the second component and 0.2 for the third component. If one component breaks down the machine will stop with probability of 0.5. If two components break down the machine will stop with probability of 0.8. If all three components break down the machine will surely stop. What is the probability that the machine will stop in a given day due to failure of its components? R: $P = 0.40$

6. The main office of a bank gets applications from its three branches. 30% of which come from branch A, 50 % from branch B and the rest from branch C. Among the applications received from branch A 20% are stockbroking cases, from branch B there are 10% and from branch C there are 40% of stockbroking cases. While checking the bank operations we randomly select an application.

   a. If the selected application is not a stockbroking case, what is the probability that the application has been sent from the branch C? R: $P = 0.148$

   b. If the selected application is a stockbroking case, what is the probability that the application has not been sent from the branch B? R: $P = 0.737$

7. At the end of the production line the quality of the products is controlled. For this purpose, a computerized system has been developed which correctly recognizes a defective product in 99% of the cases, while in 3% of the cases it recognizes the good product as defective. From past experience we know that 2% of the products are defective. A product is randomly selected.

   a. If the selected product is good, what is the probability that it is recognized as good? R: $P = 0.97$

   b. If the selected product is recognized as good, what is the probability that it is defective? R: $P = 2 \cdot 10^{-4}$

Additional problems for tutorials in Random phenomena course – 1st set

Events and probability

1. At the faculty 25 % of students fail the exam in mathematics, 15 % of students fail the exam in physics and 10 % of students fail both exams.

   a. What is the probability that a randomly chosen student passes exam in mathematics and fails physics? R: $P = 0.05$

   b. What is the probability that a randomly chosen student passes both exams? R: $P = 0.70$

2. In the table, 150 traffic accidents are categorized according to their consequences and causes.

		Consequences	
		Death	Wounds
Cause	Speed	24	16
	Alcohol	46	14
	Other	30	20

   a. What is the probability that for a randomly chosen accident alcohol was not the cause and there were no victims? R: $P = 0.240$

   b. What is the probability that for a randomly chosen accident with no victims neither the alcohol nor inappropriate speed were the cause of it? R: $P = 0.400$

3. One third of products are made on machine A, half of them are made on machine B and the rest are made on machine C. 8 % of products from machine A, 14 % of products from machine B and 10 % of products from machine C are of poor quality. A product is randomly selected.

   a. What is the probability that a randomly selected product is of poor quality? R: $P = 0.113$

   b. If a randomly selected product is of poor quality, what is the probability that it was made on machine A? R: $P = 0.236$

4. Three missiles are shot at a plane. The probabilities of hitting a plane are 0.5, 0.6 and 0.8 for the first, second and the third shot, respectively. A plane that is hit only once is shot down with probability 0.3, a plane that is hit two times is shot down with probability 0.6 and a plane that is hit three times is shot down for sure. What is the probability that a plane is going to be shot down after three shots? R: $P = 0.594$

5. It has been statistically found out that 41 % of people have blood type A, 9 % of people have blood type B, 4 % of people have blood type AB and the rest have blood type 0. On the notes of the bags with donated blood mistakes occur. For 88 % of donors with blood type A notes are correct, while for 4 % of donors with blood type B, 10 % of donors with blood type AB and 4 % of donors with blood type 0 notes are for blood type A instead of correct blood type.

   a. What is the probability that a randomly chosen donor has blood type A and his/her bag has a correct note? R: $P = 0.361$

   b. What is the probability that in a randomly chosen bag with a note for blood type A there actually is blood of type A? R: $P = 0.933$

6. Medical experts developed a new test for finding a virus of some incurable disease, by which every one in ten thousands of citizens of Slovenia is infected. The test gives positive result in 99.9 % of cases of infection, while the test is negative in 99 % of cases when the person is not infected. As a citizen of Slovenia you have been tested and the result is positive. What is the probability that you are actually not infected? R: $P = 0.990$

7. Among tested products which can only have one type of defect 30 % of products have defect A, 50 % of products have defect B and the rest of the products have some other defects. 30 % of products with a defect A do not function, among products with a defect B there are 10 % of them that do not function and among other product there are 20 % of products that do not function. A product is randomly selected.

   a. What is the probability that the selected product functions? R: $P = 0.820$

   b. If the selected product functions, what is the probability that it has defect A or defect B? R: $P = 0.805$