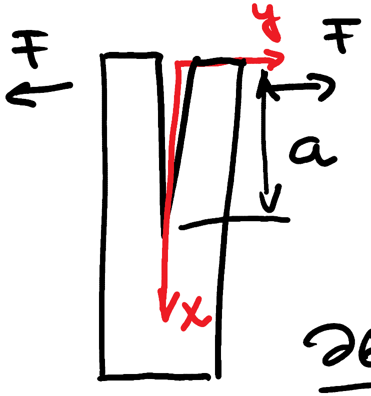
RAST UTRUJENOSTNIH RAZPOK

DOVOLJENO NA IZPITU

MODUS I TIP RAZPOKE



PREDPOSTAVKE: ELASTIČNI MATERIALNI MODEL
HOMOGEN IN IZOTROPEN MATERIAL
RAZPOKE NISO ZELO MAJHNE

POTREBUJEMO:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0$$

RAVNOTEŽNE ENAČBE ZA NAPETOSTI

$$E \varepsilon_x = \sigma_x - \nu \sigma_y$$

$$E \varepsilon_y = \sigma_y - \nu \sigma_x$$

$$G \gamma_{xy} = \tau_{xy}$$

HOOKOV ZAKON

$$G = \frac{E}{2(1+\nu)}$$

$$\varepsilon_x = \frac{\partial u}{\partial x} \; ; \quad \varepsilon_y = \frac{\partial v}{\partial y} \; ; \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

KOMPATIBILNOSTNE ENAČBE

$$\frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \frac{\partial^2}{\partial y^2}\left(\frac{\partial u}{\partial x}\right) + \frac{\partial^2}{\partial x^2}\left(\frac{\partial v}{\partial y}\right) = \frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2}$$

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2} \; ; \quad \sigma_y = \frac{\partial^2 \phi}{\partial x^2} \; ; \quad \tau_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y}$$

AIRYJEVA FUNKCIJA ϕ

$$\frac{\partial^4 \phi}{\partial y^4} + \frac{\partial^4 \phi}{\partial x^4} + \frac{2\partial^4 \phi}{\partial x^2 \partial y^2} = 0$$

$$\phi = \mathrm{Re}\,\bar{\bar{Z}}(z) + y\,\mathrm{Im}\,\bar{Z}(z)$$

$$\bar{Z}(z) = \frac{d\bar{\bar{Z}}(z)}{dz} \; ; \quad Z(z) = \frac{d\bar{Z}(z)}{dz}$$

WESTERGARDOVA FUNKCIJA $Z(z)$

DOVOLJENO NA IZPITU

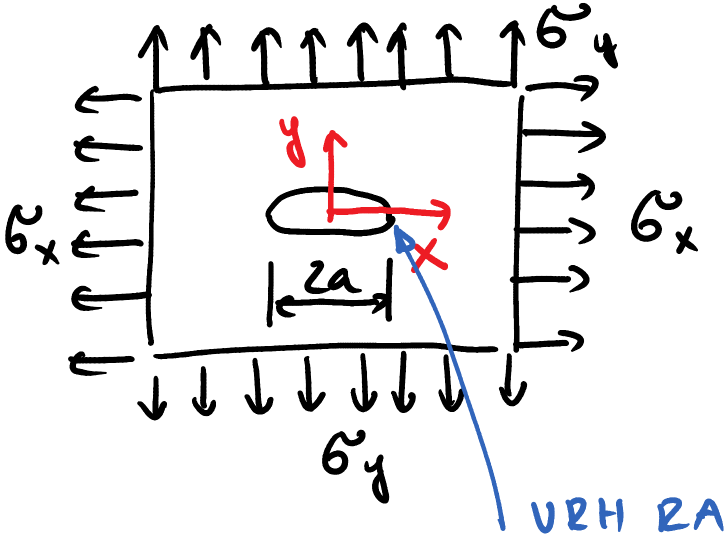
$$\frac{\partial \operatorname{Re} f(z)}{\partial x} = \frac{\partial \operatorname{Im} f(z)}{\partial y} = \operatorname{Re} \frac{\partial f(z)}{\partial z}$$

$$\frac{\partial \operatorname{Im} f(z)}{\partial x} = -\frac{\partial \operatorname{Re} f(z)}{\partial y} = \operatorname{Im} \frac{\partial f(z)}{\partial z}$$

CAUCHY-RIEMANOVA
PRAVILA ZA ODVAJANJE

DOVOLJENO NA
IZPITU

NAPETOSTNO STANJE OB VRHU RAZPOKE



DVOOSNO OBREMENJENA
PLOŠČA S
CENTRALNO RAZPOKO DOLŽINE 2a

$$\sigma_x = \sigma_y = \sigma$$

——— DEJANSKA OBLIKA RAZPOKE

NAPETOST σ_y

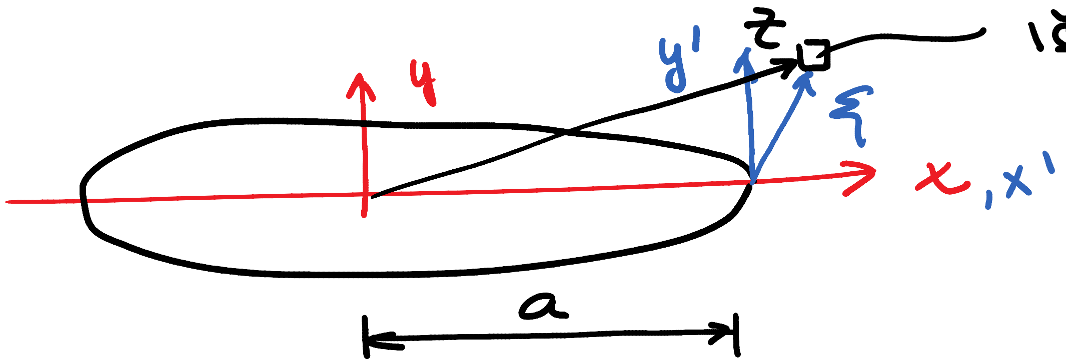
$$\sigma_y = \frac{\partial^2 \phi}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial \operatorname{Re} \bar{\bar{Z}}(z)}{\partial x} + y \frac{\partial \operatorname{Im} \bar{Z}(z)}{\partial x}\right)$$

$$= \frac{\partial}{\partial x}\left(\operatorname{Re} \frac{\partial \bar{\bar{Z}}(z)}{\partial z} + y \operatorname{Im} \frac{\partial \bar{Z}(z)}{\partial z}\right)$$

$$= \frac{\partial}{\partial x}\left(\operatorname{Re} \bar{Z}(z) + y \operatorname{Im} Z(z)\right)$$

$$= \frac{\partial \operatorname{Re} \bar{Z}(z)}{\partial x} + y \frac{\partial \operatorname{Im} Z(z)}{\partial x} = \operatorname{Re} \frac{\partial \bar{Z}(z)}{\partial z} + y \operatorname{Im} \frac{\partial Z(z)}{\partial z}$$

$$\sigma_y = \operatorname{Re} Z(z) + y \operatorname{Im} Z'(z)$$



IŠČEMO NAPETOST

$$z = a + \xi$$

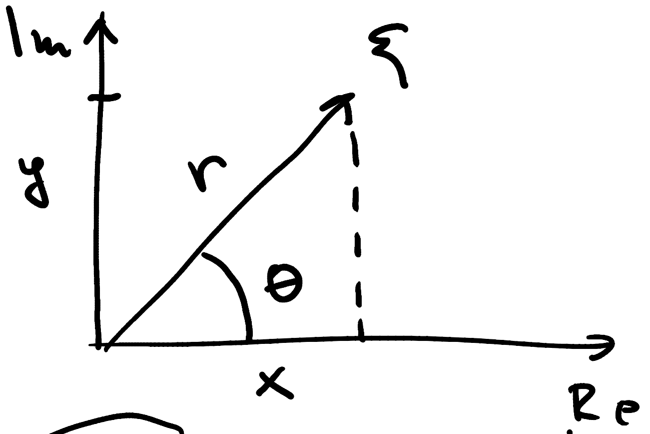
$$a \gg \xi$$

$$Z(z) = \frac{\sigma_0 \cdot z}{\sqrt{z^2 - a^2}} = \frac{\sigma_0 (a + \overset{0}{\cancel{\xi}})}{\sqrt{\cancel{a^2} + 2a\xi + \underset{0}{\cancel{\xi^2}} - \cancel{a^2}}} = \frac{\sigma_0 a}{\sqrt{2a\xi}}$$

$$= \frac{\sigma_0 \sqrt{a}}{\sqrt{2\xi}} = Z(a+\xi)$$

$$Z'(z) = Z'(\xi) = \frac{\sigma_0 \sqrt{a}}{\sqrt{2}} \left(-\frac{1}{2}\right) \xi^{-\frac{3}{2}} = -\frac{\sigma_0 \sqrt{a}}{\sqrt{2\xi}\; 2\xi}$$

$$\mathrm{Re}\, z(z) = \mathrm{Re} \frac{\sigma\sqrt{a}}{\sqrt{2\xi}}$$



$$\xi = x + iy$$

$$r = |\xi|$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$\xi = r(\cos\theta + i\sin\theta) = r e^{i\theta}$$

EULERJEVA FORMULA

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\mathrm{Re} \frac{\sigma\sqrt{a}}{\sqrt{2r}} e^{-\frac{i\theta}{2}} =$$

$$\frac{\sigma\sqrt{a}}{\sqrt{2r}} \cos\left(-\frac{\theta}{2}\right) = \frac{\sigma\sqrt{a}}{\sqrt{2r}} \cos\frac{\theta}{2}$$

$$\mathrm{Im}\, z'(z) = -\mathrm{Im}\,\frac{\sigma^{\infty}\sqrt{a}}{\sqrt{2\xi}\,2\xi} = -\mathrm{Im}\,\frac{\sigma^{\infty}\sqrt{a}}{\sqrt{2r}\,2r}\, e^{-\frac{i3\theta}{2}}$$

$$= -\frac{\sigma^{\infty}\sqrt{a}}{\sqrt{2r}\,2r}\sin\left(-\frac{3\theta}{2}\right) = \frac{\sigma^{\infty}\sqrt{a}}{\sqrt{2r}\,2r}\sin\frac{3\theta}{2}$$

$$\sigma_y = \frac{\sigma^{\infty}\sqrt{a}}{\sqrt{2r}}\left(\cos\frac{\theta}{2} + y\,\frac{1}{\cancel{2r}}\sin\frac{3\theta}{2}\right)$$

$$\overset{\|}{\cancel{r}\sin\theta} = \cancel{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\sigma_y = \frac{\sigma^{\infty}\sqrt{\pi a}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$ ■

NAPETOST σ_x

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial \mathrm{Re}\,\bar{\bar{Z}}(z)}{\partial y} + \mathrm{Im}\,\bar{Z}(z) + y\,\frac{\partial \mathrm{Im}\,\bar{Z}(z)}{\partial y} \right)$$

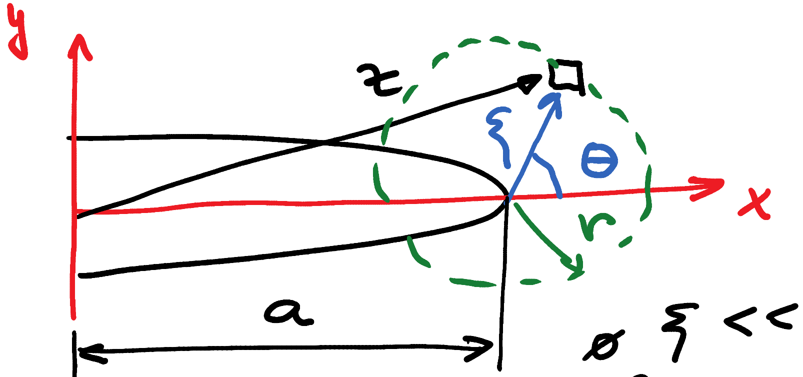
$$\phi = \mathrm{Re}\,\bar{\bar{Z}}(z) + y\,\mathrm{Im}\,\bar{Z}(z)$$

$$\sigma_x = \frac{\partial}{\partial y}\left(-\mathrm{Im}\,\frac{\partial \bar{\bar{Z}}(z)}{\partial z} + \mathrm{Im}\,\bar{Z}(z) + y\,\mathrm{Re}\,\frac{\partial \bar{Z}(z)}{\partial z} \right)$$

$$\sigma_x = \frac{\partial}{\partial y}\left(-\cancel{\mathrm{Im}\,\bar{Z}(z)} + \cancel{\mathrm{Im}\,\bar{Z}(z)} + y\,\mathrm{Re}\,Z(z) \right)$$

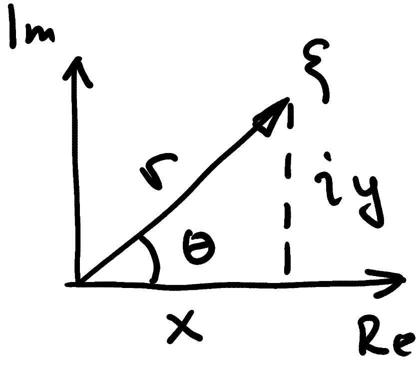
$$\sigma_x = \mathrm{Re}\,Z(z) + y\,\frac{\partial \mathrm{Re}\,Z(z)}{\partial y} = \mathrm{Re}\,Z(z) - y\,\mathrm{Im}\,Z'(z)$$

$$Z(z) = \frac{\sigma}{\sqrt{1-\frac{a^2}{z^2}}}$$



$$0 \quad \xi << a$$

$$z = a + \xi$$



$$Z(z) = \frac{\sigma z}{\sqrt{z^2 - a^2}} = \frac{\sigma(a + \cancel{\xi})}{\sqrt{\cancel{a^2} + 2a\xi + \underset{0}{\cancel{\xi^2}} - \cancel{a^2}}}$$

$$\xi = x + iy$$

$$\xi = r e^{i\theta}$$

$$x = r \cos\theta$$

$$y = r \sin\theta$$

$$Z(z) = Z(\xi) = \frac{\sigma a}{\sqrt{2a\xi}} = \frac{\sigma\sqrt{a}}{\sqrt{2\xi}}$$

$$Z'(z) = Z'(\xi) = \frac{\sigma a}{\sqrt{2a}\ \xi^{\frac{3}{2}}}\left(-\frac{1}{2}\right)$$

$$= \frac{-\sigma\sqrt{a}}{\sqrt{2\xi}\ 2\xi}$$

$$\mathrm{Re}\, z(z) = \mathrm{Re}\, z(\xi) = \mathrm{Re}\, \frac{\sigma\sqrt{a}}{\sqrt{2\xi}} = \mathrm{Re}\, \frac{\sigma\sqrt{a}}{\sqrt{2r}}\, e^{-i\frac{\theta}{2}}$$

$$= \mathrm{Re}\, \frac{\sigma\sqrt{a}}{\sqrt{2}}$$

$$\xi^{\frac{1}{2}} = (r\, e^{i\theta})^{\frac{1}{2}} = r^{\frac{1}{2}}\, e^{\frac{i\theta}{2}}$$

$$\cancel{r}\, e^{\frac{i\theta}{2}} = \cancel{r} \cos\frac{\theta}{2} + i\,\cancel{r} \sin\frac{\theta}{2}$$

$$e^{-\frac{i\theta}{2}} = \cos\frac{\theta}{2} - i\sin\frac{\theta}{2}$$

$$\mathrm{Re}\, z(z) = \frac{\sigma\sqrt{a}}{\sqrt{2r}}\cos\frac{\theta}{2}$$

$$\mathrm{Im}\, z'(z) = \mathrm{Im}\, z'(\xi) = -\mathrm{Im}\, \frac{6\sqrt{a}}{\sqrt{2q}\, 2\xi} = -\mathrm{Im}\, \frac{6\sqrt{a}}{\sqrt{2r}\, 2r}\, e^{-\frac{i\theta 3}{2}}$$

$$\xi^{\frac{3}{2}} = (r e^{i\theta})^{\frac{3}{2}} = r^{\frac{3}{2}}\, e^{\frac{i\theta 3}{2}}$$

$$e^{-\frac{i\theta 3}{2}} = \cos\frac{3\theta}{2} - i\sin\frac{3\theta}{2}$$

$$\mathrm{Im}\, z'(z) = \frac{6\sqrt{a}}{\sqrt{2r}\, 2r}\, \sin\frac{3\theta}{2}$$

$$y\, \mathrm{Im}\, z'(z) = \cancel{r} \sin\theta\, \frac{6\sqrt{a}}{\sqrt{2r}\, 2\cancel{r}}\, \sin\frac{3\theta}{2}$$

$$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$y\, \mathrm{Im}\, z'(z) = \frac{6\sqrt{a}}{\sqrt{2r}}\, \cos\frac{\theta}{2}\, \sin\frac{\theta}{2}\, \sin\frac{3\theta}{2}$$

$$\sigma_x = \frac{\sigma\sqrt{a}}{\sqrt{2r}} \underbrace{\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)}_{f(\theta)}$$

ODVISNA OD OBREMENITVE IN GEOMETRIJE RAZPOKE

$$\sigma_x = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}} f(\theta)$$

$$\sigma_y = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$\theta = 0$

$$\sigma_x = \sigma_y = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}} \quad ; \quad \tau_{xy} = 0$$

$K = \sigma\sqrt{\pi a}$ FAKTOR INTENZIVNOSTI NAPETOSTI

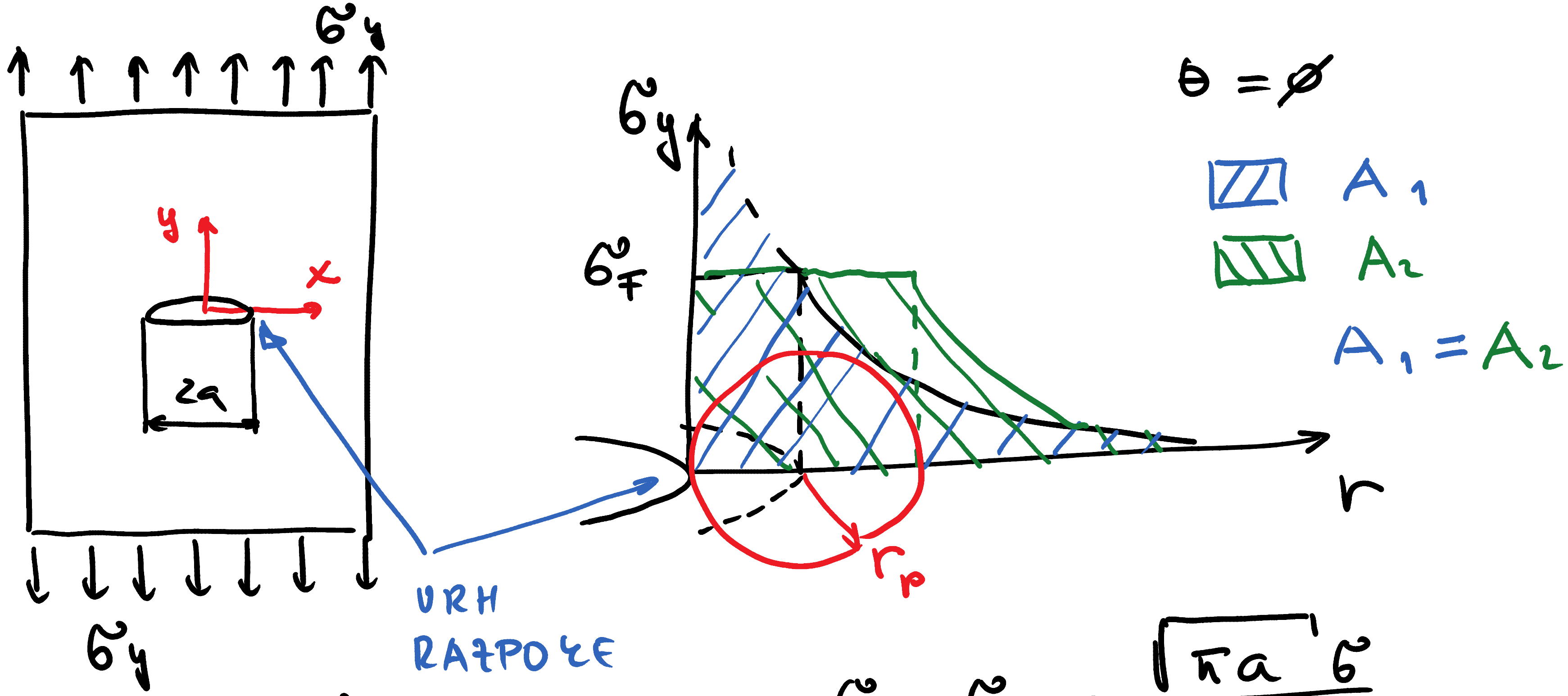
$K = \beta_1 \beta_2 \sigma \sqrt{\pi a}$

FAKTOR INTENZIVNOSTI NAPETOSTI
Z UPOŠTEVANJEM UPLIVA
OBREMENITVE β_1 IN
GEOMETRIJE RAZPOKE β_2

↑

ŠE VEDNO JE PREDPOSTAVLJENO
ELASTIČNO NAPETOSTNO-DEFORMACIJSKO
STANJE!

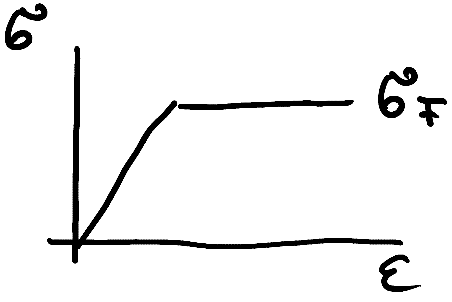
VPLIV PLASTIČNOSTI NA FAKTOR INTENZIVNOSTI NAPETOSTI



$\theta = 0$

$\sigma_y = \sigma$

$\sigma_x = 0$



$$\sigma_y = \sigma_F = \frac{\sqrt{\pi a}\,\sigma}{\sqrt{2\pi r_p}}$$

$$r_p = \frac{a\,\sigma^2}{2\,\sigma_F^2}$$

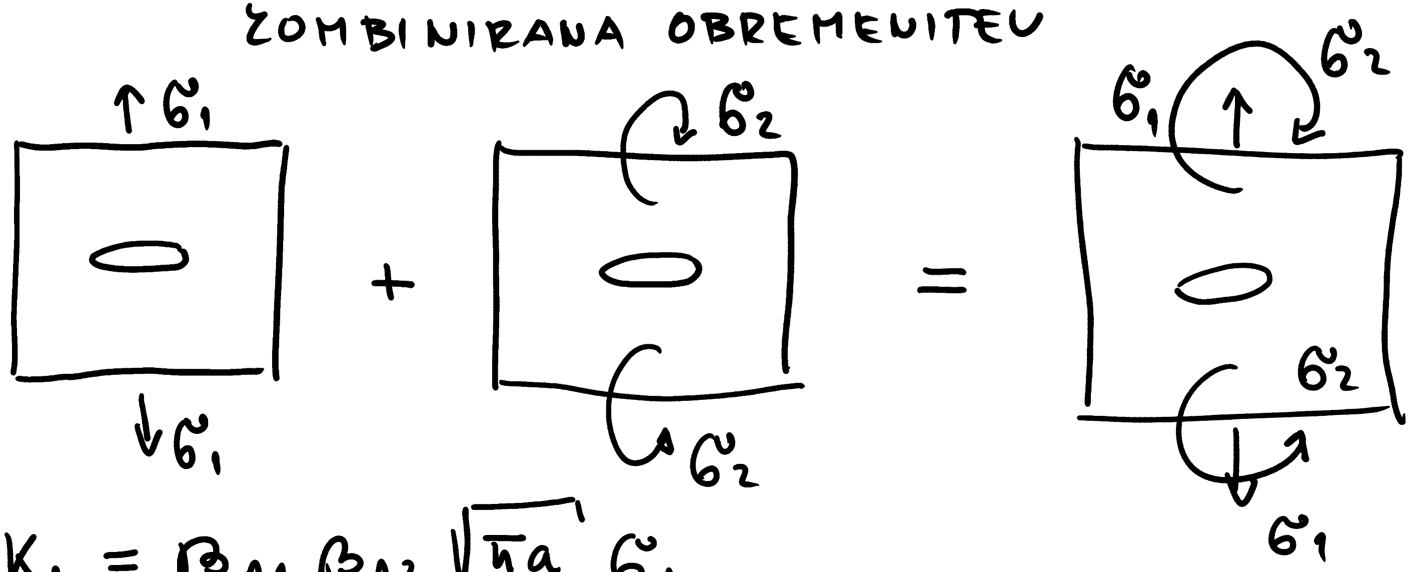
$$K = \beta_1 \cdot \beta_2 \sqrt{\pi (a + r_p)}\ \sigma$$

$$\underset{\uparrow}{a} = \underset{\uparrow}{a} + r_p$$

DOLŽINA RAZPOKE BREZ UPOŠTEVANJA PLASTIČNOSTI

DOLŽINA RAZPOKE Z UPOŠTEVANJEM PLASTIČNOSTI

KOMBINIRANA OBREMENITEV



$$K_1 = \beta_{11}\,\beta_{12}\sqrt{\pi a}\;\sigma_1$$

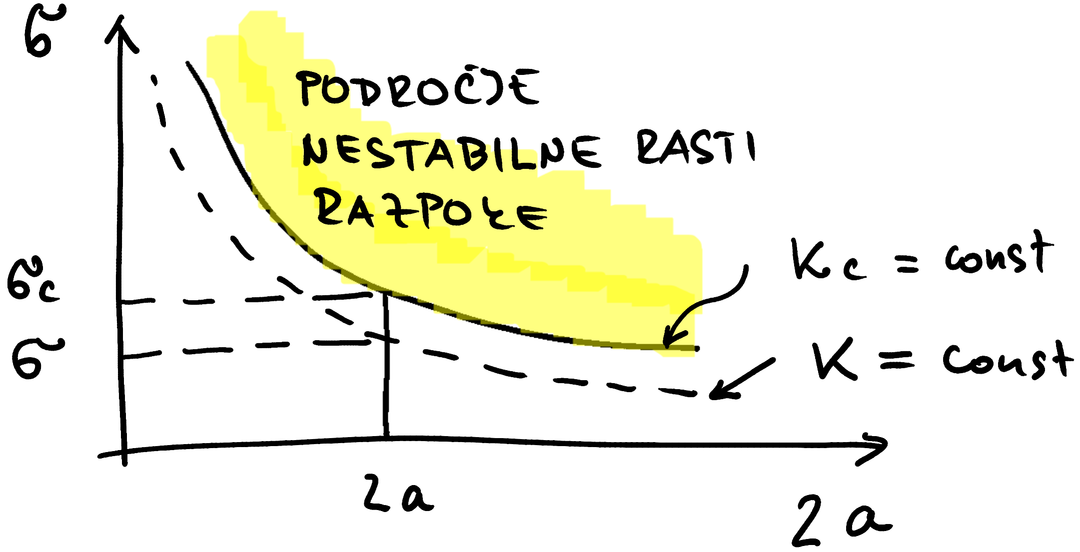
$$K_2 = \beta_{21}\,\beta_{22}\sqrt{\pi a}\;\sigma_2$$

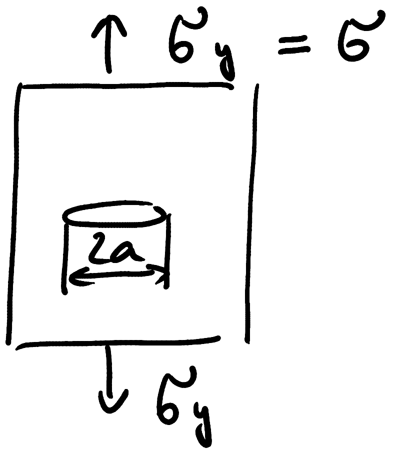
$$K = K_1 + K_2$$

$$K = \sqrt{\pi a}\;\sum_i \beta_{i1}\,\beta_{i2}\,\sigma_i$$

KRITIČNI FAKTOR INTENIVNOSTI NAPETOSTI

$K_c = \beta_1 \beta_2 \sigma_c \sqrt{\pi a}$ IMENUJEMO LOMNA TRDNOST





$$\sigma = \frac{K}{\beta_1 \beta_2 \sqrt{\pi a}}$$

ZA HCF

$\sigma \le R_m$ IN LCF

$K \le K_c$

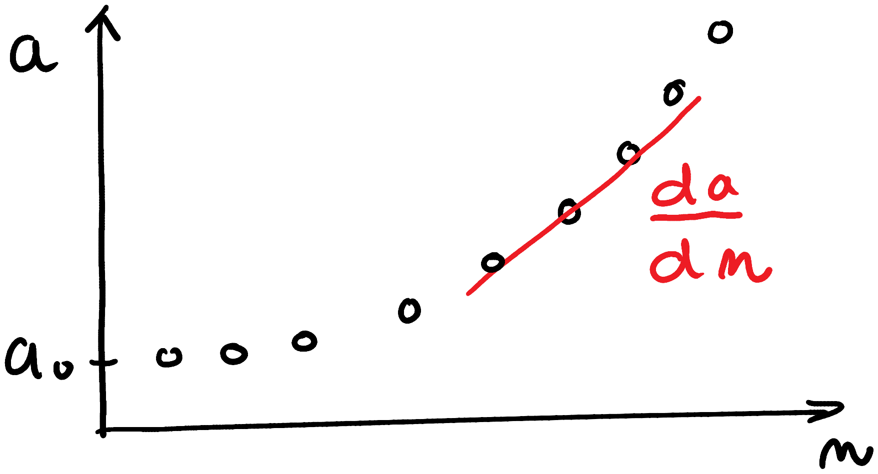
PARISOVA ENAČBA

$$\frac{da}{dn} = C(\Delta K)^m$$

$$\Delta K = K_{max} - K_{min}$$

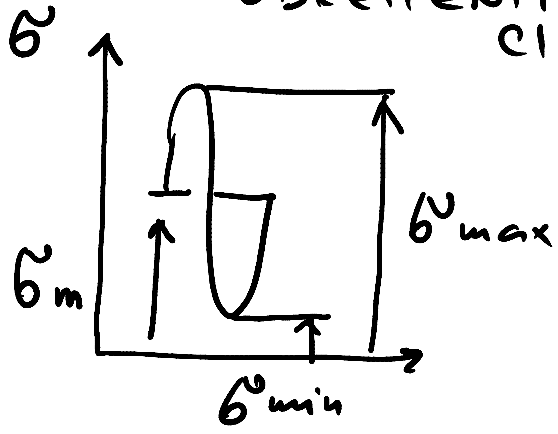
$$K_{max} = \beta_1 \beta_2 \sigma_{max} \sqrt{\pi a}$$

$$K_{min} = \beta_1 \beta_2 \sigma_{min} \sqrt{\pi a}$$



ΔK RANG FAKTORJA INTENZIVNOSTI NAPETOSTI

OBREMENITVENI CIKEL



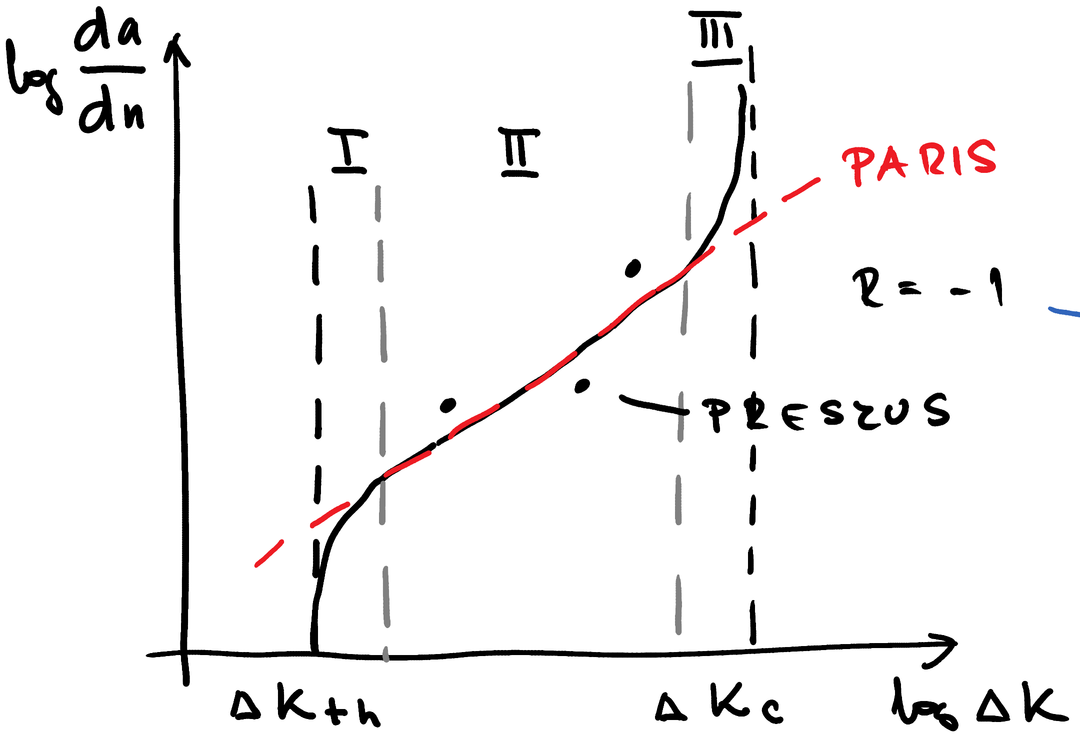
n ŠTEVILO OBREMENITVENIH CIKLOV

ZANEMARIMO VPLIV σ_m

a_0 ZAČETNA VELIKOST RAZPOKE

$$\log \frac{da}{dn} = \log C + m \log \Delta K$$

$$y = \log C + m \cdot x$$



$R = -1$

K_{th} ≡ TRAJNI
DINAMIČNI
TRDNOSTI

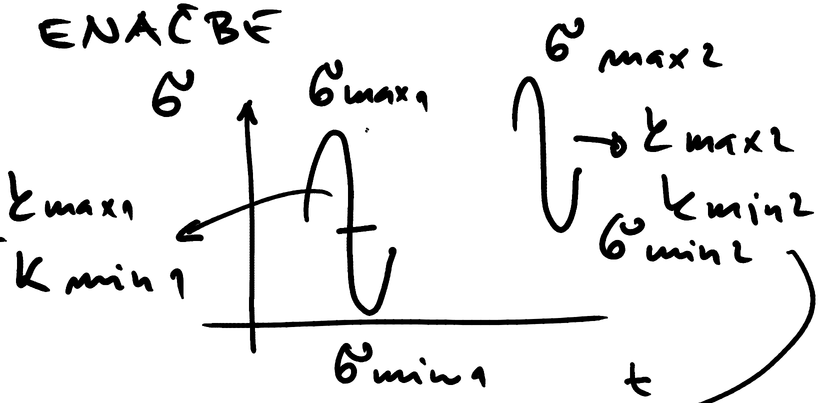
MEJNA VREDNOST
FAKTORJA
INTENZIVNOSTI
NAPETOSTI

$\Delta K_c = 2 K_c$

$\Delta K_{th} = 2 K_{th}$

PRAKTIČNA UPORABA PARISOVE ENAČBE

$$\frac{da}{dn} \rightarrow \frac{\Delta a}{\underbrace{\Delta n}_{1}} = C\,\Delta K^{m}$$



$$a_0$$

$$a_1 = a_0 + \Delta a_1 = a_0 + C\,\Delta K_1^{m}$$

$$a_2 = a_1 + \Delta a_2 = a_1 + C\,\Delta K_2^{m} = a_0 + C\,\Delta K_1^{m} + C\,\Delta K_2^{m}$$

$$\vdots$$

$$a_i = a_0 + \sum_{j=1}^{i} C\,\Delta K_j^{m}$$

$$\Delta K_2 = \beta_1 \beta_2 \sqrt{\pi a_1}\,(\sigma_{max2} - \sigma_{min2})$$

$$\Delta K_1 = \beta_1 \beta_2 \sqrt{\pi a_0}\,(\sigma_{max1} - \sigma_{min1})$$

$$a_i \leq a_{dop}$$