2. EXERCISE

Vertical response of a vehicle when driving over the obstacle

Vehicle dynamics

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1. Task definition

Define the vertical response of a two-axle vehicle in the case when a vehicle at a specified speed v (in the table) drives over the obstacle of the prescribed shape (in the table). To determine the response, construct a half-model of the vehicle, taking into account the mass of the chassis, the stiffness of the suspension, the damping of the shock absorber, the mass of the axles and the stiffness of the contact with the ground. Get the technical data for a particular type of the vehicle yourself.

Group	Vehicle type	Speed	Obstacle shape
1	Personal vehicle	25 km/h	Bulge: half sinus, H=10cm; L=50cm
2	Personal vehicle	15 km/h	Bulge: half sinus, H=14cm; L=100cm
3	Personal vehicle	20 km/h	Bulge: half sinus, H=14cm; L=50cm
4	Personal vehicle	30km/h	Bulge: half sinus, H=8cm; L=50cm
5	Personal vehicle	40 km/h	Bulge: half sinus, H=10cm; L=10cm
6	Personal vehicle	50 km/h	Bulge: half sinus, H=14cm; L=200cm
7	Personal vehicle	60 km/h	Bulge: half sinus, H=10cm; L=50cm
8	Personal vehicle	30 km/h	Bulge: half sinus, H=14cm; L=100cm
9	Personal vehicle	45 km/h	Bulge: half sinus, H=12cm; L=70cm
10	Personal vehicle	25 km/h	Bulge: half sinus, H=16cm; L=80cm
11	Commercial vehicle	30 km/h	Bulge: half sinus, H=14cm; L=50cm
12	Commercial vehicle	40 km/h	Bulge: half sinus, H=8cm; L=50cm
13	Commercial vehicle	50 km/h	Bulge: half sinus, H=10cm; L=10cm
14	Commercial vehicle	25 km/h	Bulge: half sinus, H=14cm; L=200cm
15	Commercial vehicle	35 km/h	Bulge: half sinus, H=10cm; L=50cm
16	Commercial vehicle	40 km/h	Bulge: half sinus, H=10cm; L=50cm
17	Commercial vehicle	20 km/h	Bulge: half sinus, H=14cm; L=100cm

For a particular vehicle type draw a simplified dynamic model of the vehicle and derive the differential equations for the vertical direction for it. To determine the response, solve the equations numerically using the Euler method. A correctly solved task with a perfect report will be rated with a maximum grade of 8. For higher grades, the actual damping in the shock absorber (shrinkage, stretching) must be taken into account, and possible jump of the vehicle over the bulge (if necessary, simulate the bulge that causes the jump).
Create a computer program independently, which must include three essential elements: the data entry unit, the vertical vehicle recalculation unit, and the output unit. If you want to avoid writing the program, you can also solve the task with any of the programs, such as EXCEL. In this case too, the spreadsheet must contain all three of the abovementioned elements. The program must be written in such a way that it allows changing the input data (in particular, the shape of the bulge and the speed of the vehicle), and when the input data changes, the vehicle's response has to change automatically.

The report has to include the calculations and results of the following:

- simplified (half) dynamic model of the vehicle with all the denotations,
- derive a system of differential equations for determining the response,
- procedure of solving the equations using Euler's approach,

- procedure for determining the excitation of the vehicle model *x(t),*
- response diagram of the first and second vehicle axis (on the same chart together with the excitation),
- response diagram of the chassis (movement, rotation and excitation on the same chart),
- common response diagram (excitation, chassis response and axles response on the same chart).

For higher grades further show:

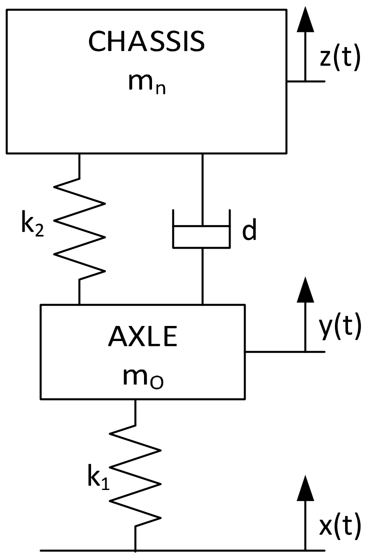
- the method of taking into account the different damping in the shock absorber during shrinkage or stretching,
- the method for considering the jump of the vehicle,
- a diagram of the chassis and axles responses taking into account the vehicle's jump.

The report must show the resolution process with all of the equations you have used. Refer to sources. Together with the report it is also necessary to submit the following:

- computer program in electronic version,
- for the ability to control your program, also prepare a file / table of input data used,
- presented calculation should be performed using the data (values) from attached input data table.

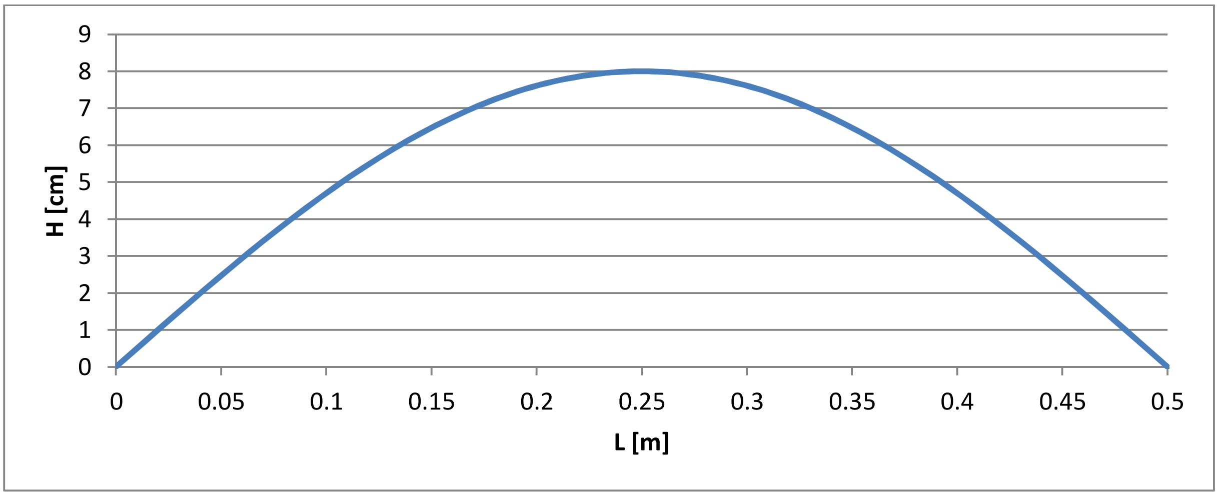
2. Example of a quarterly vehicle dynamic model

2.1 Physical model



2.2 Excitation determination

Excitation of the system *x(t)* depends on the shape of the obstacle over which the vehicle drives and the vehicle speed. In case the obstacle shape is given as a half sinus (Diag. 1) the excitation can be determined following presented procedure:



Diag. 1: Obstacle (bulge) shape

$$x(t) = H \cdot \sin(\omega t) \tag{1}$$

Where ω depends on the obstacle length and vehicle speed.

The transition time across the obstacle is calculated with the Eq. (46).

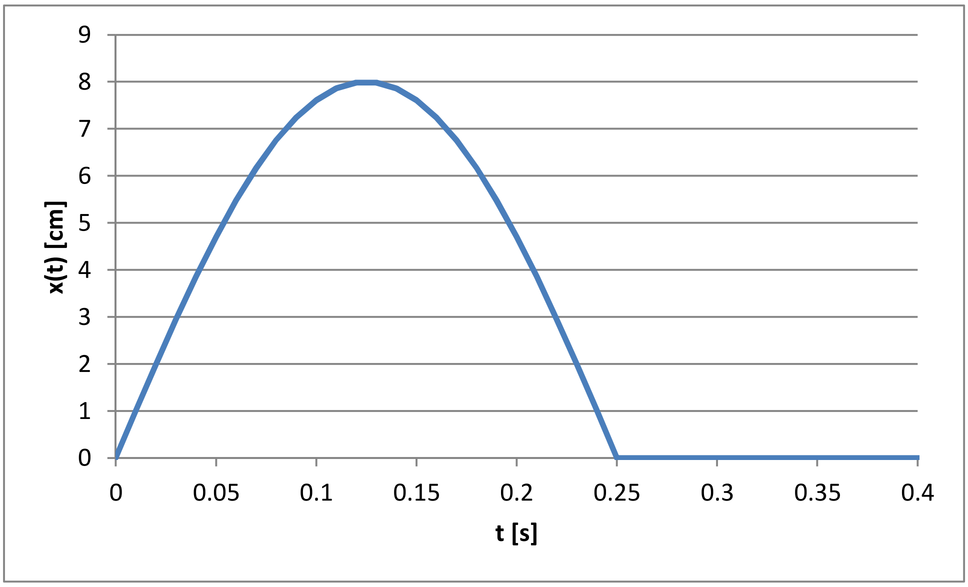
$$t_p = \frac{L}{v} \tag{2}$$

ω is determined using Eq. (47).

$$\omega = \frac{\pi}{t_p} \tag{3}$$

Eq. (45) can then be changed to:

$$x(t) = H \cdot \sin(\frac{\pi \cdot v}{L} t) \tag{4}$$



Diag. 2: Example of excitation diagram for the vehicle speed of 2m/s and the obstacle shape from Diag. 1

2.3 Derivation of a differential equation system

Chassis:

$$\sum F_{iZ} = 0$$
$$F_{spring} + F_{absorber} = m_n \cdot a_n$$
$$k_2 \cdot \big(y(t) - z(t)\big) + d \cdot \big(\dot{y}(t) - \dot{z}(t)\big) = m_n \cdot \ddot{z}(t)$$

Axle:

$$\sum F_{iZ} = 0$$
$$F_{tire} - F_{spring} - F_{absorber} = m_o \cdot a_o$$
$$k_1 \cdot \big(x(t) - y(t)\big) - k_2 \cdot \big(y(t) - z(t)\big) - d \cdot \big(\dot{y}(t) - \dot{z}(t)\big) = m_o \cdot \ddot{y}(t)$$

To determine the system responses *y(t)* and *z(t)* it is necessary to solve a system of differential equations. To do that, we can use the so-called Euler numerical method.

2.4 Euler's method for solving first order differential equations

Let's deal with the differential equation of the first order:

$$\frac{dy(t)}{dt} + a \cdot y(t) = f(t) \text{ or } \frac{dy(t)}{dt} = f(t) - a \cdot y(t) \tag{5}$$

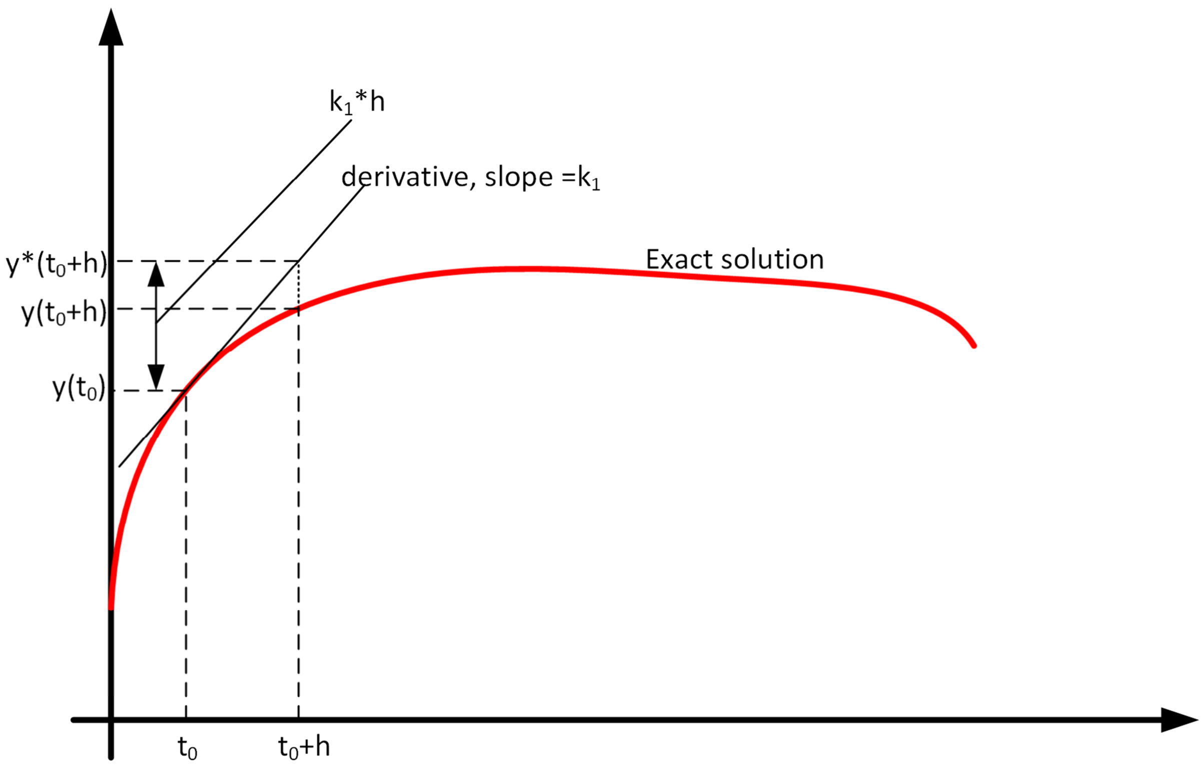
If we begin at some time t_0 the value of the function at time t_0+h ($y(t_0+h)$) can be approximated by the value of $y(t_0)$ to which we add the product of the time step h and the slope of the function defined by the derivative.

$$y(t_0 + h) \approx y(t_0) + h \cdot \left.\frac{dy(t)}{dt}\right|_{t=t_0} \tag{6}$$

This value can be called the approximate value of the function and is denoted by $y^*(t)$. To calculate the approximate value we need to know the value of the function at the start time t_0.

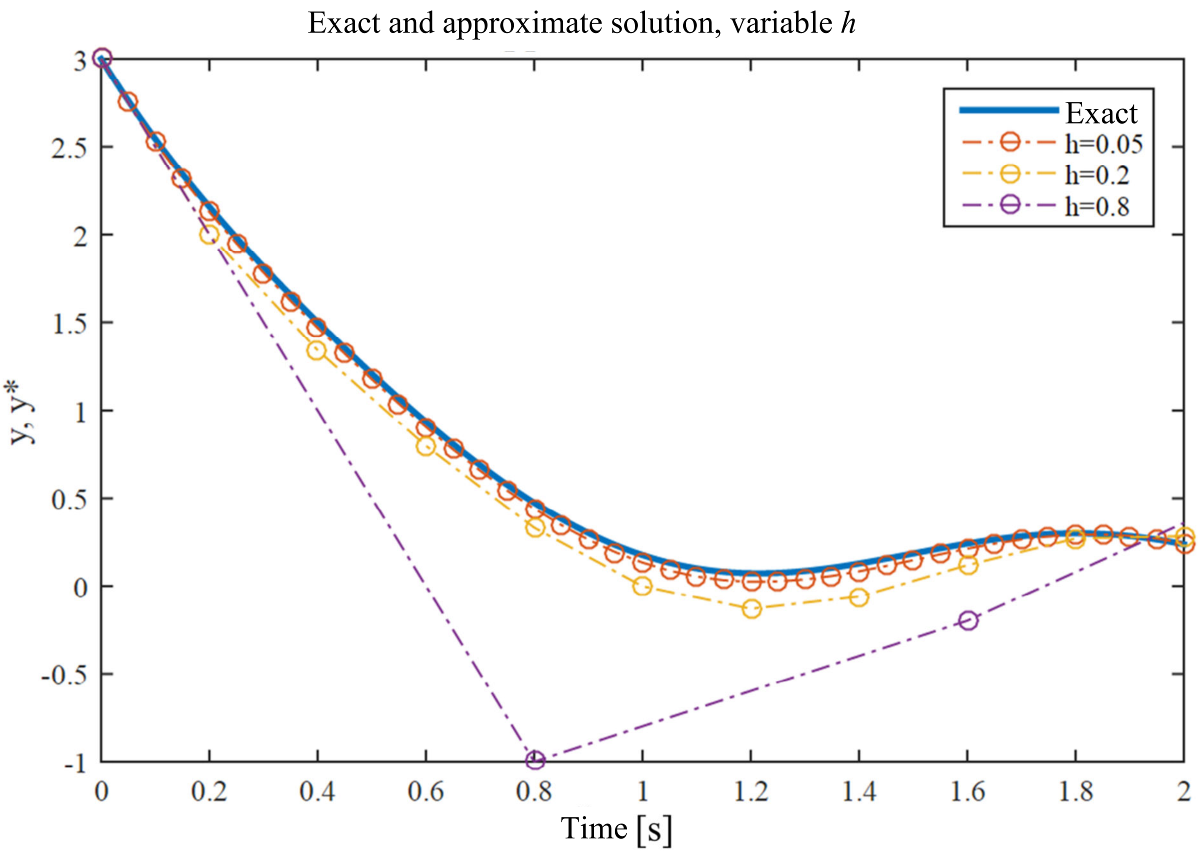
$$y^*(t_0 + h) \approx y(t_0) + h \cdot \left.\frac{dy(t)}{dt}\right|_{t=t_0} \tag{7}$$

If we can calculate the value of the derivative dy/dt at time t_0 with the Eq. (49) then with Eq. (50) we can calculate the approximation value of y at time t_0+h. The obtained value $y^*(t_0+h)$ can further be used for the calculation of the derivative dy/dt at time t_0+h and so on until we reach the desired time.



Diag. 3: A graphical representation of Euler's method

Diag. 13 graphically shows the use of the Euler method where the derivative at t_0 is denoted as k_1. From the diagram, it is obvious that the success of this method strongly depends on the length of the time step h. If the time step h is too large then we can expect the error to be large as well. For this reason, the time step should be selected appropriately. The influence of the time step length on the accuracy of the solution is also well seen in Diag. 4.



Diag. 4: The influence of the time step length on the solution

2.5 Euler's method for solving higher order differential equations

In order to solve differential equations of higher order, we can also use the Euler's numerical method described in the previous chapter. To do that, it is necessary to break the higher-order equation into several first-order equations, which are then solved according to the described procedure.
Let's deal with the following differential equation:

$$\frac{d^2y(t)}{dt^2} + 100 \cdot \frac{dy(t)}{dt} + 10^4 \cdot y(t) = 10^4 \cdot |\sin(377t)| \tag{8}$$

We will assume that all initial conditions are the same and are 0.

The analytical solution of the equation would be very difficult, but for the Euler method it does not pose a particular problem. The first thing that needs to be done is to rearrange the problem to differential equations of the first order. We introduce two new variables $x_1(t)$ in $x_2(t)$ where $x_1(t)=y(t)$. Now we can write down two connected differential equations of the first order:

$$\frac{dx_1(t)}{dt} = x_2(t) \qquad \left(= \frac{dy(t)}{dt}\right) \quad \text{and} \tag{9}$$

$$\begin{aligned} \frac{dx_2(t)}{dt} &= -10^4 \cdot y(t) - 100 \cdot \frac{dy(t)}{dt} + 10^4 \cdot |\sin(377t)| \\ &= -10^4 \cdot x_1(t) - 100 \cdot x_2(t) + 10^4 \cdot |\sin(377t)| \end{aligned} \tag{10}$$

For the simultaneous solution of both equations, the Euler method can be used in accordance with the following procedure.

- Beginning at time t_0, we first need to determine the length of the time step h and find the initial conditions for all variables (in the present case $x_1(t_0)$ and $x_2(t_0)$).
- By means of the initial values $x_i(t_0)$ we can calculate the derivatives for each of the variables $x_i(t)$ at time $t=t_0$. These values can be denoted as k_{1i}.

$$k_{1i} = \left.\frac{dx_i(t)}{dt}\right|_{t_0}; \quad k_{11} = \left.\frac{dx_1(t)}{dt}\right|_{t_0}, k_{12} = \left.\frac{dx_2(t)}{dt}\right|_{t_0} \tag{11}$$

- From these values we can determine the approximate values for each $x_i^*(t_0 + h)$.

$$x_i^*(t_0 + h) = x_i(t_0) + k_{1i} \cdot h \tag{12}$$

- In the next step we define new start time $t_0 = t_0 + h$ and for each variable $x_i(t_0)$ we calculate $x_i(t_0) = x_i^*(t_0 + h)$.
- We repeat the procedure (points 2-4) until solution is known.