1. EXERCISE

Dynamic characteristics of vehicles

Vehicle dynamics

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Contents

1. Task definition

Select an arbitrary road vehicle with an internal combustion engine, for which you can find the relevant input data in accessible literature. Create an application which must include three essential elements: a data entry unit, a calculation unit, and an output unit. If you want to avoid writing the program code, you can also solve the task with any of the programs, such as EXCEL. In this case the spreadsheet must contain all three of the abovementioned elements.

The report has to include the calculations and results of the following characteristics:

- diagram of the external characteristics of the engine $P_M(n)$ and $M_M(n)$,
- diagram of the vehicle traction forces $F_K(v)$ and resistances $R(v)$,
- diagram of dynamic coefficient $D(v)$,
- diagram of time, path and acceleration as a function of speed $a(v)$, $t(v)$ in $s(v)$,
- diagram of power balance $P_K(v)$ together with needed power to overcome individual and collective resistance $P(v)_{resistance}$,
- diagram of vehicle speed as a function of gear ratio and engine speed $v(n)$,
- a diagram of limiting slopes $\alpha(v)$
- diagram of power supply $\Delta P(v)$,
- the following has to be determined from the diagrams:
  - maximum allowable slope α,
  - maximum possible acceleration of the vehicle a,
  - needed time to accelerate from 0 to 100 km/h,
  - maximum speed of the vehicle v.

For the best grade: when determining the limit values (acceleration, slope) also consider possible slip of the tires.
The report must show the resolution process with all of the equations you have used. Refer to sources. Together with the report it is necessary to deliver:

- application in electronic version. Application has to be user friendly.
- To simplify the accuracy control of your results, provide a table with all input data used in the report.

1.1 *Example of vehicle data*

Vehicle: Renault Twingo II 1.2

Tab. 1: External characteristics of the engine

rpm [1/min]	Power (corr) [kW]	Torque [Nm]
1518,00	11,70	73,80
2015,00	16,50	78,30
2305,00	20,00	82,80
2506,00	22,30	84,80
2710,00	24,00	84,50
3011,00	25,90	82,20
3510,00	30,70	83,60
4005,00	34,80	83,00
4499,00	37,70	80,00
5002,00	38,30	73,10
5251,00	37,90	68,80
5493,00	36,50	63,40
5999,00	32,30	51,40

Tab. 2: Weights and loads

	Empty vehicle	Max. load
Load of front axle	6060 N	8100 N
Load of rear axle	4040 N	5400 N
Total vehicle weight	10100 N	13500 N

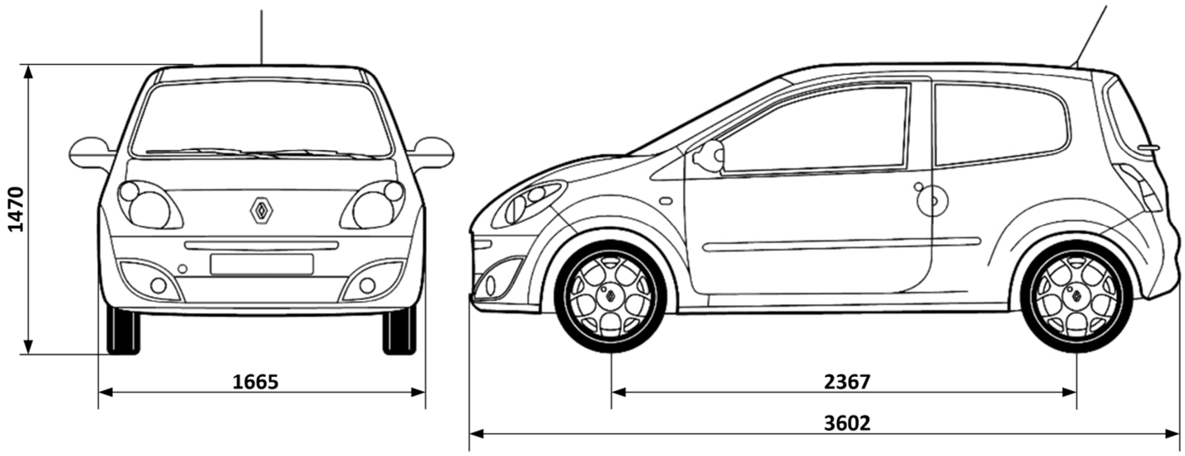


Figure 1: Vehicle sketch with main dimensions

Tab. 3: Gear rations of the gearbox and differential plus efficiency

	denotation	Gear ratio	denotation	Transmission efficiency
First gear	i_I	3,73	η_I	0,98
Second gear	i_{II}	2,05	η_{II}	0,98
Third gear	i_{III}	1,39	η_{III}	0,99
Fourth gear	i_{IV}	1,03	η_{IV}	0,99
Fifth gear	i_V	0,80	η_V	0,99
Reverse gear	i_{vz}	3,55	η_{vz}	0,98
Differential gear ratio	i_{dif}	3,56	/	/
Efficiency of other transmissions	/	/	η_{ost}	0,90

Other data:

- Rolling resistance factor - f = 0,01
- Tire dimensions 185/55 - R15
- Cross section area of the vehicle - A = 2,2m^2
- Drag coefficient of the vehicle - c = 0,35

2. Theoretical background of the calculation

The purpose of this exercise is to address the dynamic driving conditions, such as determining the necessary power to overcome certain driving conditions, determining acceleration, acceleration times, maximum speed etc.

2.1 Basic structure of the vehicle

In this exercise, we are interested in the dynamic characteristics of the vehicle. Therefore, we will not be concerned with the structure and properties of the bodywork of the vehicle which apart from its weight and the drag coefficient has no major influence on the dynamic properties of the vehicle. Attention will be paid to the drive assembly, the diagram of which is shown in Figure 2.

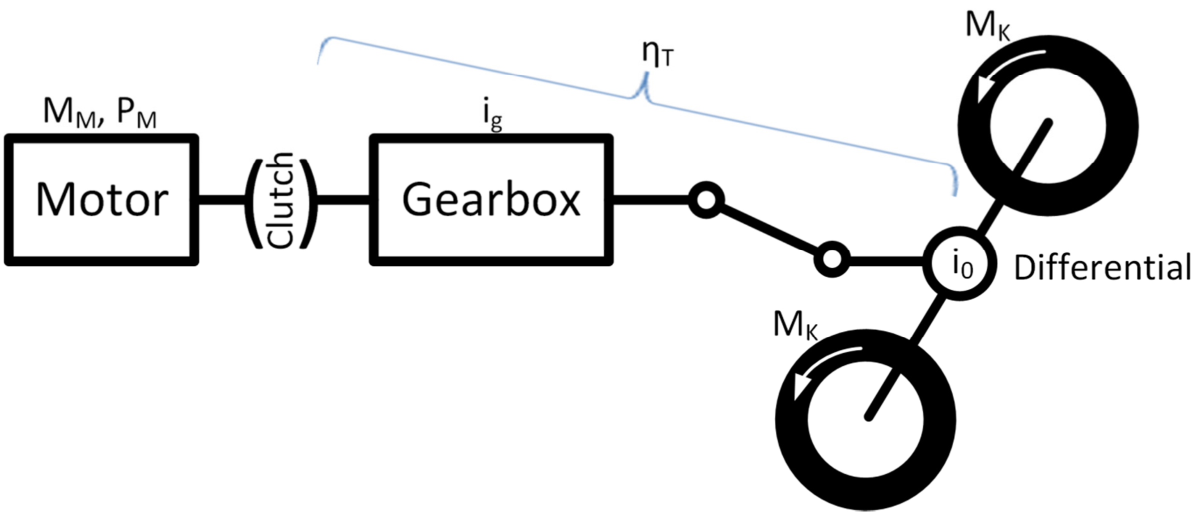


Figure 2: Sketch of the vehicle drive assembly

The vehicle is schematically illustrated by an engine that provides a certain torque at a given speed of the crankshaft. Torque is then transmitted over the clutch to the gearbox and then through the differential to the wheels.

2.2 Motor

The motor serves to convert the stored energy into mechanical rotational energy. The stored energy can be in the form of chemical energy such as gasoline, gas oil, coal, hydrogen, etc., or in the form of electricity stored in batteries. Depending on the type of energy stored, several types of motors are distinguished:

- internal combustion engines
- steam engine
- gas turbine
- electric motor

Internal combustion engines have mostly been used as propulsion machines in vehicles and electric motors have been increasing in recent years. In this exercises we will focus on vehicles with internal combustion engines.

The engine as the propulsion machine in the vehicle should meet some requirements:

- the output shaft of the engine should have a variable speed
- at each speed of the output shaft, it should have the same (maximum) output power.

Today's internal combustion engines partially meet the first requirement, while they do not meet the second one. The typical characteristic of an internal combustion engine is shown in Figure 3.

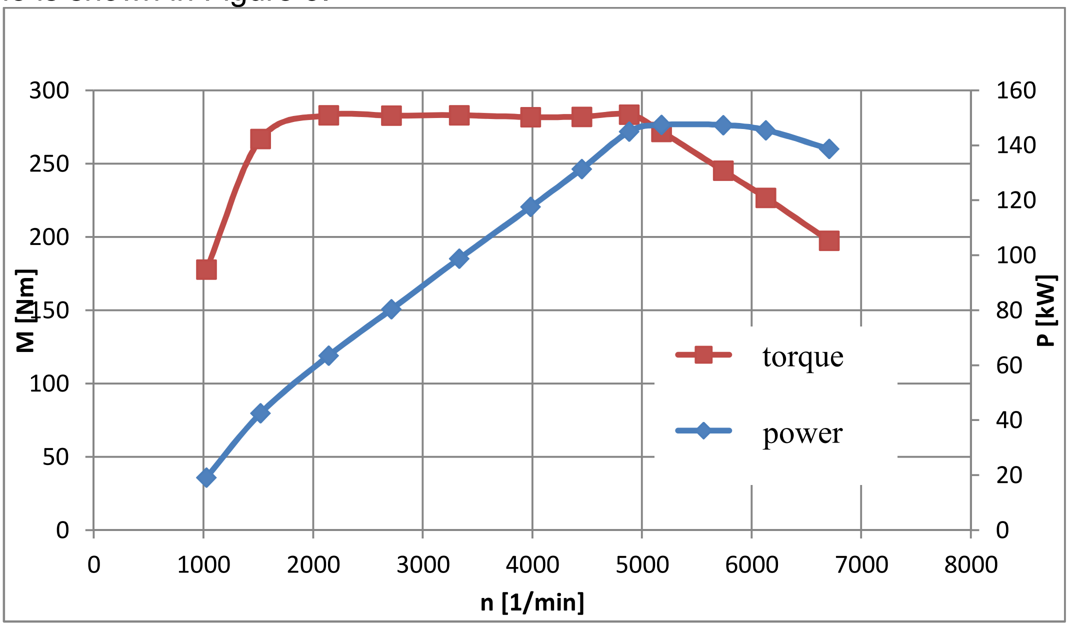


Figure 3: Example of the internal combustion engine external characteristics

Internal combustion engines are required to have a main usable area with as wide a range of crankshaft speeds as possible. The most up-to-date engines fulfil this requirement through modern technical solutions such as computer-controlled fuel injection, changing the times of opening and closing valves, changing the lengths of the suction tubes, etc.

The engine torque can be expressed depending on the engine power and engine speed, which is indicated by eq. (1) [1].

$$M_M = \frac{P_M}{\omega_M} \quad [\text{Nm}] \text{ or } M_M = \frac{30 \cdot P_M}{\pi \cdot n_M} \quad [\text{Nm}] \tag{1}$$

where:

- P_M [W] engine power
- M_M [Nm] engine torque
- n_M [min^{-1}] speed of the crankshaft

- ω_M [rad/s] angular velocity of the crankshaft

2.3 Gearbox

Usually it is desirable to keep as much power on the wheels as possible. As already described, today's engines do not allow this on a sufficiently large speed range (0 - v_{max}). The demand for constant power on the drive wheels can be accomplished by the use of a continuously variable transmission, but in practice, the stage gearbox is used in practice, which is a satisfactory solution. Used stage gearboxes usually have five or more forward gears and one reverse gear. In the lower gears, this gearbox acts as a reducer, and in the higher (from 4th onwards), often as a multiplier with a gear ratio i_i.

2.4 Differential

When traveling through the corner the speed of the inner drive wheel is smaller than the speed of the outer wheel due to different trajectory radii. Therefore the wheels must not be rigidly connected to one another. The torque is then transmitted from the gearbox to the wheels via the differential, which allows different revolutions of the outer and inner wheels. The differential unit also serves as a gearbox with a gear ratio i_{kg}.

2.5 Wheels

Wheels serve to transfer forces from vehicles to the road. The wheel consists of a rim and a tire. To determine the dynamics of a vehicle, it is important to know the dynamic and static radius (r_d, r_{st}) of the tire and the coefficient of friction between the tire and the road.
When the data on the size of the dynamic radius of the wheel is not known (which in most cases is a fact), eq. (2) can be used to roughly calculate it from the dimensions of the tire.

$$r_{st} \approx r_d = \frac{D[''] \cdot 25{,}4[mm/'']}{2} + \frac{b[mm] \cdot x[\%]}{100[\%]} \tag{2}$$

where:

- r_{st} [mm] ... static radius of the wheel
- r_d [mm] ... dynamic radius of the wheel
- D [″] .... radius of the rim
- b [mm] ... tire width (section width)
- x [%] ...... sidewall aspect ratio

2.6 Dynamic driving conditions

2.6.1 Kinematics of the vehicle

The engine as a propulsion machine in the vehicle operates only in the specified speed range of the crankshaft (n_{min}-n_{max}). Therefore the devices described in the previous section are needed to overcome this issue. The speed of the vehicle is therefore dependent on the engine speed and the selected gear ratio in the gearbox.

The speed of the vehicle is determined by the Eq. (3) and the speed of the wheels with the Eq. (4).

$$v = \frac{n_k \cdot \pi \cdot r_d}{30} \tag{3}$$

where:

- v [m/s].... vehicle speed
- n_k [min^{-1}] . rotational speed of the wheels

$$n_k = \frac{n_m}{i_i \cdot i_{kg}} \tag{4}$$

where:

- i_i [/] .... current gear ratio
- i_{kg} [/] .... gear ratios of the differential

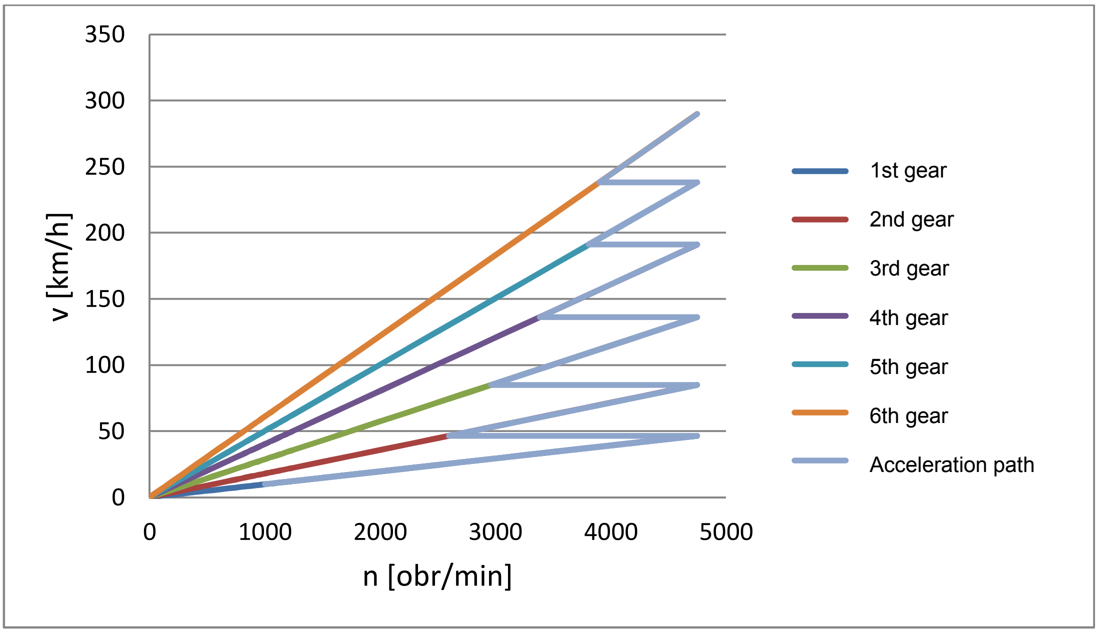


Figure 4: Diagram of vehicle speed as a function of gear ratio and engine speed

Figure 4 shows the vehicle speed as a function of used gear and speed of the crankshaft. The "Acceleration path" curve shows the acceleration path where shift between gears is performed at maximum crank shaft speed. In the case of properly calculated gear ratios the acceleration path is always within the range determined by the speed of the maximum torque and the maximum engine power.

2.6.2 Driving resistances

Every movement on the Earth's surface is subjected with the energy loss due to different resistances. In the case of linear movement of the vehicle, the following driving resistances shall appear::

- R_f [N]**.......** rolling resistance,
- R_s [N]**.......** slope (hill) resistance,
- R_z [N]**.......** air resistance (drag),

- R_i [N]....... resistance of mass inertia and
- R_p [N]....... resistance of trailer.

2.6.2.1 Rolling resistance R_f

The rolling resistance acts in the contact area between the tires and the road surface. It is the result of the deformation of the tires and the road surface. In case of level road driving, R_f is calculated by equation

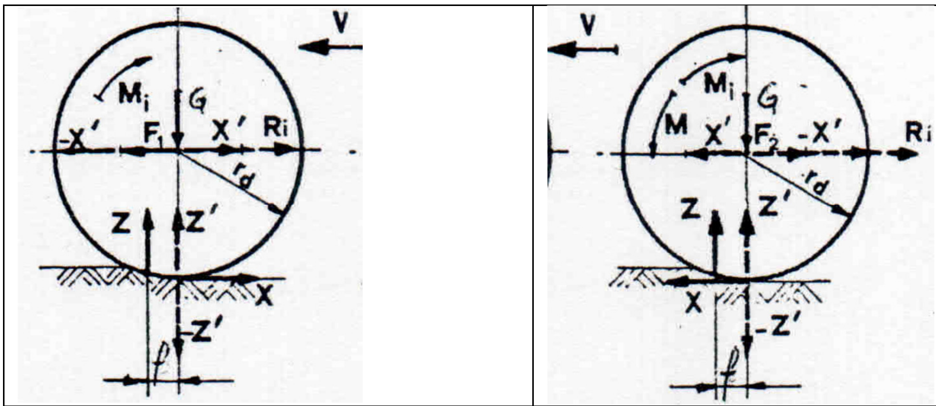
$$R_f = f \cdot \sum Z_i = f \cdot G \tag{5}$$

in the case of driving the vehicle in the slope, the rolling resistance is reduced

$$R_f = f \cdot G \cdot \cos(\alpha) \tag{6}$$

where:

- G [N]....... vehicle weight
- f [/] .... rolling resistance coefficient
- α [°]........ angle of the slope



a.) Driven wheel — b.) Driver wheel

Figure 5: Rolling resistance on driver and driven wheel

Tab. 4: Indicated values of the rolling resistance coefficient f

Road type	*f*
asphalt concrete (tarmac), smooth	0,010
concrete, smooth	0,010
concrete, rough	0,014
curbstone, very good	0,015
curbstone, good	0,020
curbstone, poor	0,033
macadam, poor	0,035
field road, very good	0,045
field road, good	0,080
field road, poor	0,160
sand, non-compacted, dry	0,150 – 0,300

Another cause for the formation of a rolling resistance is the tangential displacements in the tire's supporting surface, which cause slipping. These tangential displacements depend on the design of the tire running layer, which can be radial or diagonal. There are practically no tangential movements in the radial tires, therefore the rolling resistance is lower. The rolling resistance coefficient is determined by experiments and is a function of:

$$f = f\left(Q_{tp}, p_p, Q_c, v\right) \tag{7}$$

where:

- Q_{tp}........... [/] quality of tire running layer
- Q_c[/] .... quality of the road
- p_p [Pa]..... tire pressure
- v [m/s].... vehicle speed

When calculating, we assume that the rolling resistance does not change with vehicle speed. The rolling resistance of the vehicle at straight driving is composed of:

- basic rolling resistance of the tires,
- toe-in toe-out effect and
- resistance due to driving on uneven road.

The proportions of resistance due to the toe-in toe-out effect and due to uneven road are usually very small and can therefore be neglected.

2.6.2.2 Air resistance (drag) R_z

The air resistance consists of the following components:

- pressure resistance resulting from all normal pressure forces acting on the surface of the vehicle, or resistor of the shape,,
- friction resistance, which is the result of all tangential forces acting on the surface of the vehicle, or the resistance of the surface,,
- resistance that occur as a result of essential parts of the vehicle (locks, mirrors, ...) which in any way deviate from the basic vehicle profile and
- resistance resulting from the flow of air through the engine cooler and through the interior of the vehicle.

R_z in case the speed of the air w = 0 is calculated using equation:

$$R_z = \frac{1}{2} \cdot \rho \cdot A \cdot c \cdot v^2 \tag{8}$$

where:

- ρ [kg/m^3] .....air density
- c [/] ....drag coefficient that includes all above mentioned influences ($c = c_1+c_2+c_3+c_4$)
- A [m^2]..........the surface obtained as a vehicle's projection to a plane perpendicular to the direction of motion; this is the so-called front surface
- v [m/s].........vehicle speed

- w [m/s].........absolute value of air speed

In case when the speed of the air is not zero, the relative vehicle speed is calculated taking into account the speed of the air w ($v = v \pm w$).
The front surface of the vehicle is calculated using equation:

- for personal vehicles

$$A \approx 0.9 \cdot B \cdot H \qquad (9)$$

- for commercial vehicles

$$A \approx 0.78 \cdot B \cdot H \qquad (10)$$

where:

- B [m] ...... vehicle width
- H [m] ...... vehicle height

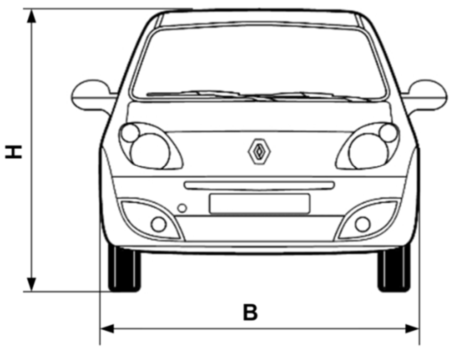


Figure 6: Vehicle's projection to a plane perpendicular to the direction of motion

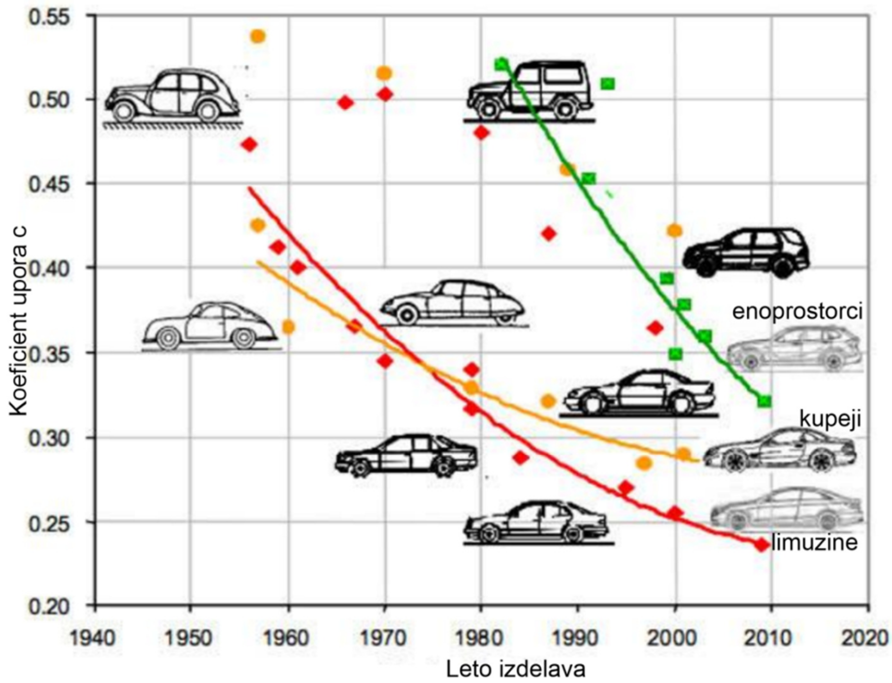


Figure 7: Drag coefficient for different vehicle types

2.6.2.3 Slope resistance R_S (resistance of hill)

The weight component parallel to the slope R_s is called a slope resistance or an ascent resistance. From the parallelogram of forces it is clear that it is defined as

$$R_S = \pm G \cdot \sin(\alpha) \tag{11}$$

Usually the angle α is defined in %. This corresponds to the tangent of the angle between the slope and the horizontal plane

$$\tan\alpha = \frac{\alpha_{\%}}{100} \tag{12}$$

The resistance of the slope can be positive or negative, depending on the direction of travel. The slope resistance brakes the vehicle when driving uphill ($R_s<0$), and accelerate the vehicle when driving downhill ($R_s>0$).

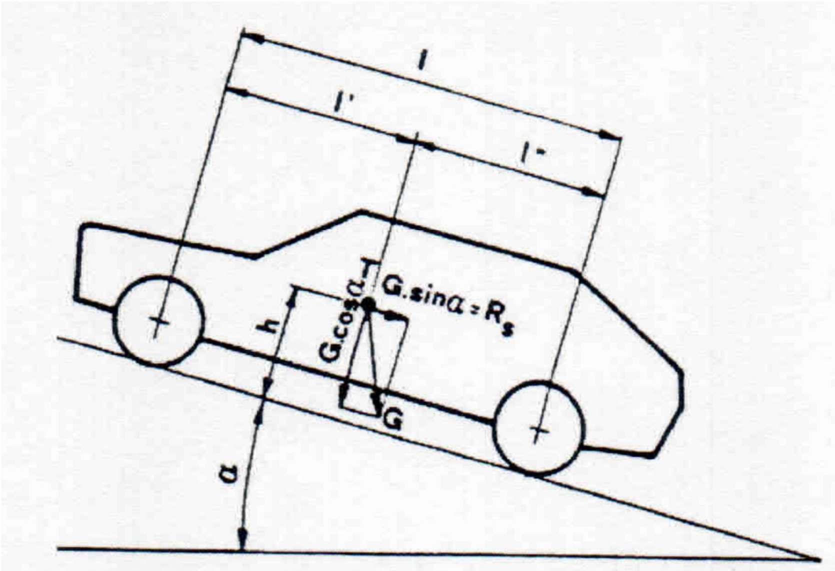


Figure 8: Slope resistance

When designing vehicles, we take into account the largest allowable road inclination in Europe, which is 26%. For vehicles intended for special use, however, the maximum ascent should be further defined.

2.6.2.4 Resistance of mass inertia R_i

At accelerated movement a part of the power is used to accelerate translatory masses R_i', and the other part to accelerate the vehicle's rotational masses R''_i [2].

$$R_i = R_i^{'} + R_i^{''} \tag{13}$$

$$R_i^{'} = \frac{G}{g} \cdot a \quad ; \quad R_i^{''} = (J_m \cdot \frac{i_m^2 \cdot i_{kg}^2}{r_d^2} \cdot \eta + z \cdot \frac{J_k}{r_d^2}) \cdot a \tag{14}$$

Total resistance of mass inertia is therefore [2]

$$R_i = \frac{G}{g} a (1 + J_m \frac{i_m^2 i_{kg}^2}{r_d^2 m} \eta + z \frac{J_k}{r_d^2 m}) \tag{15}$$

$$R_i = R_i^{'} \cdot \delta = \frac{G}{g} \cdot a \cdot \delta \tag{16}$$

where:

- δ [/] .... coefficient of rotational masses

Coefficient of rotational masses δ cannot be calculated due to unknown mass inertia of all rotating parts but can be estimated using equation (17) [2].

$$\delta = 1.03 + k \cdot i_m^2 \tag{17}$$

where:

- k **=** 0.04 (personal vehicles) ÷ 0.07 (commercial vehicles)
- i_m …gear ratio

Coefficient of rotational masses can also be estimated using experiential equation (Eq. (18)) [2].

$$\delta = 1 + k_1 + k_2 \cdot i_i^2 \tag{18}$$

where:

- k_1 [/] .... coefficient of rotational masses of wheels
- k_2 [/] .... coefficient of rotational masses of engine

$$k_1 = \frac{z_k \cdot I_k \cdot g}{r_{st} \cdot r_d} \tag{19}$$

where:

- z_k [/] .... number of wheels
- I_k [kgmm^2] mass inertia of the wheel
- r_{st} [mm] ... static radius of the wheel
- r_d [mm] ... dynamic radius of the wheel

$$k_2 = \frac{I_m \cdot i_{kg}^2 \cdot g}{\eta_t \cdot r_{st} \cdot r_d} \tag{20}$$

where:

- I_m [kgmm^2] average mass inertia of rotating parts of the engine
- η_t [/] .... transmission efficiency

Values of coefficients k_1 and k_2 can also be estimated based on experience:

$k_1 \approx 0{,}076$
$k_2 \approx 0{,}007$.

2.6.3 Traction forces on the wheels

The torque, which is available on the engine shaft at a certain engine speed, is transmitted through the transmission to the wheels. Value of the torque on wheels can be calculated by Eq. (21). To overcome the forces of driving resistance, a traction force is required on the wheels, which is calculated by Eq. (22) [2].

$$M_{K,i}(v) = M_M(n_m(v)) \cdot i_i \cdot i_{kg} \cdot \eta_i \cdot \eta_{kg} \cdot \eta_o = M_M\left(i_i\, i_{kg} \frac{30\,v}{\pi\, r_d}\right) \cdot i_i \cdot i_{kg} \cdot \eta_i \cdot \eta_{kg} \cdot \eta_o \tag{21}$$

where:

- $M_{K,i}(v)$...... [Nm] torque on the wheels in ith gear

- $M_M(v)$ .... [Nm] torque of the engine
- η_i [/]............. efficiency of transmission in i[th] gear
- η_{kg} [/]....... efficiency of differential
- η_o [/] .... efficiency of other transmissions

$$F_{k,i}(v)=\frac{M_{K,i}(v)}{r_d} \tag{22}$$

where:

- $F_{K,i}(v)$....... [N] traction force on the wheels in i[th] gear

In case of ideal motor (constant power at each speed) or continuous variable transmission with efficiency 1, the traction force on the wheels would be ideal. Ideal traction force can be calculated by Eq. (23).

$$F_{id}(v)=\frac{P_{konst\,(\max)}}{v} \tag{23}$$

where:

- $F_{id}(v)$ .... [N] ideal traction force on the wheels
- P_{konst}........ [W] ideal constant power of the engine

The traction forces F_K on the wheels are opposed by the driving resistances

$$F_K = \sum R = R_f + R_z + R_s + R_i \tag{24}$$

The equation is called a motion equation, or a balance of forces. Using this equation, we can calculate the total traction force F_k needed to overcome the sum of driving resistances or the amount of traction force used to overcome a given force of resistance. This equation is used in assessing vehicle driving characteristics.

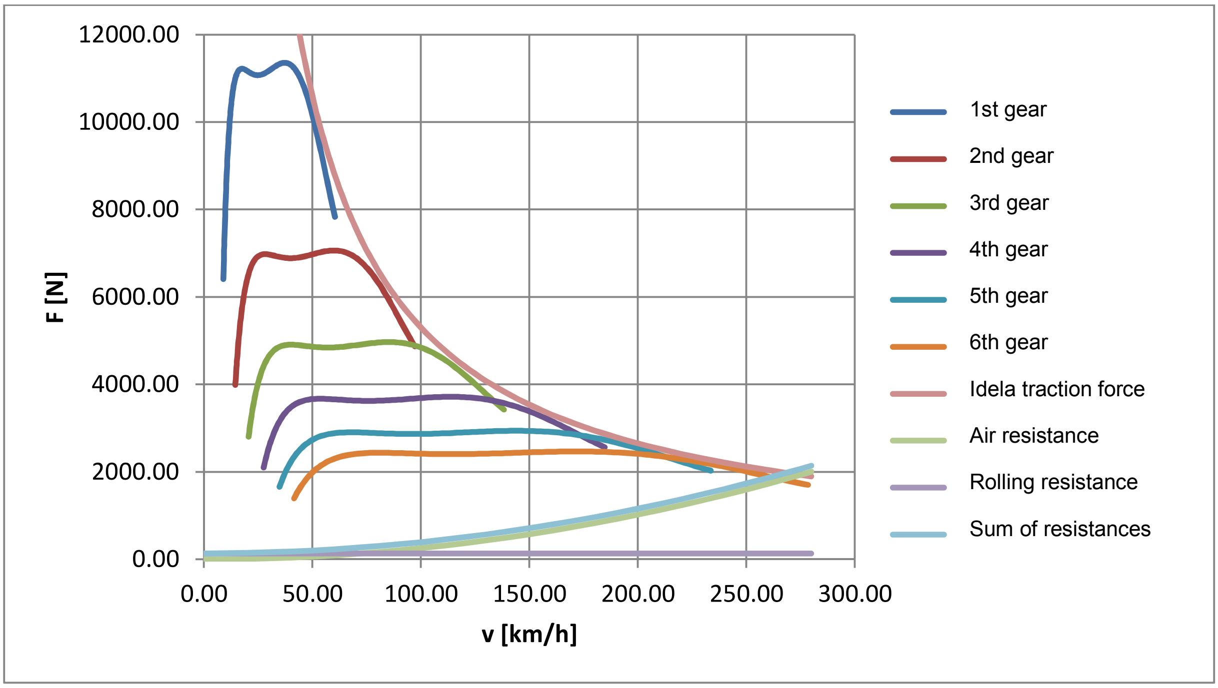


Figure 9:: Diagram of traction forces and driving resistances for vehicle VW Golf GTI

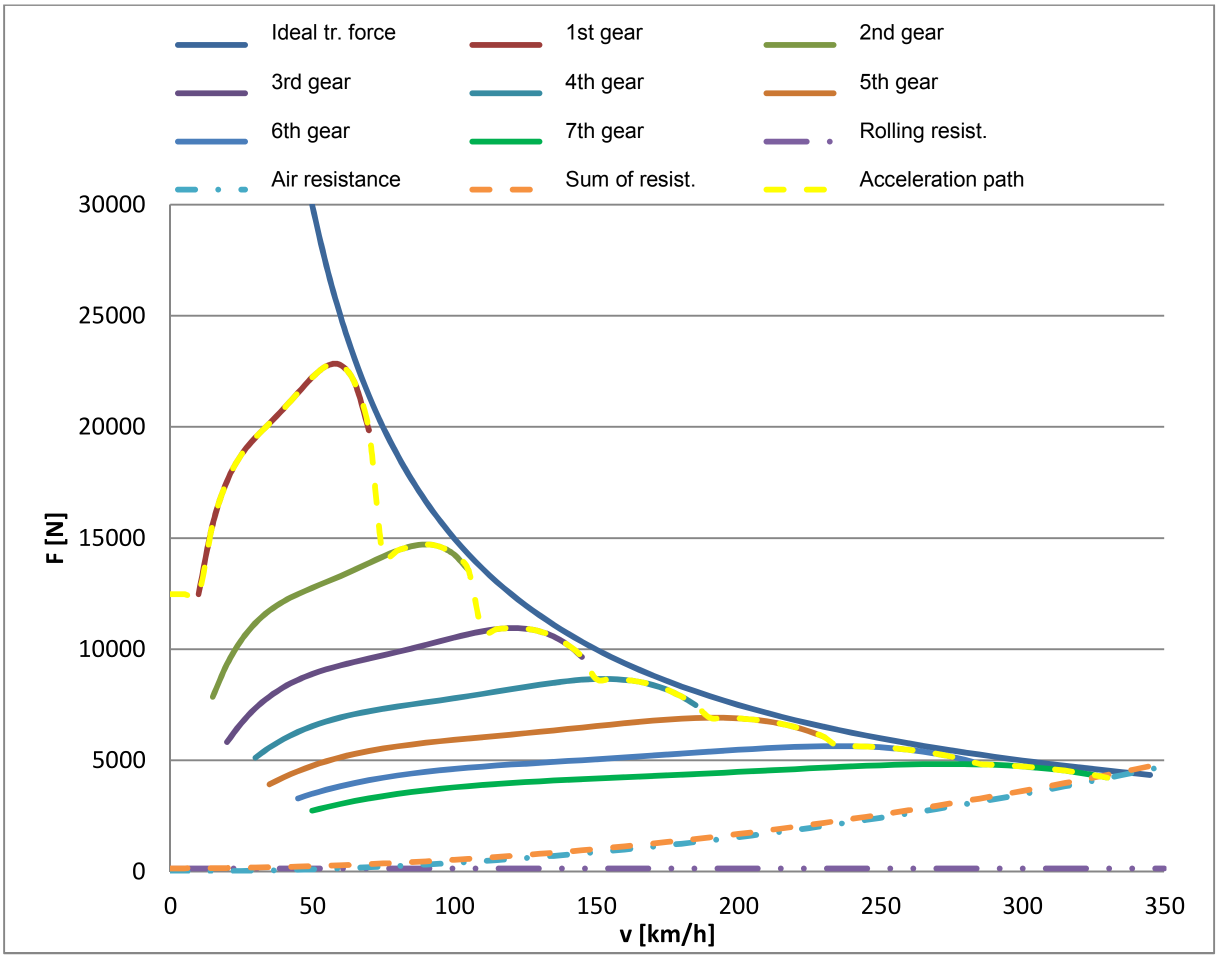


Figure 10: Diagram sil of traction forces and driving resistances for vehicle Mercedez SLS

Figure 9 and Figure 10**Error! Reference source not found.** show the size of individual forces depending on vehicle speed. From this chart maximum vehicle speed can be determined. Maximum vehicle speed is located at the intersection of the sum of resistances curve and the curve of traction force (the one that intersecst the sum of resistances curve at the highest speed $F_{(vmax)}$).

2.6.4 Dynamic coefficient of a vehicle

At a certain vehicle speed, the maximum traction force on the wheels that is opposed by the sum of resistances can be determined. The difference in the maximum traction force and the driving resistance forces is the reserve of the traction force F_{rez}, which can be used for acceleration. In order to facilitate comparison between different vehicles, a dynamic coefficient (*D*) is calculated according to Eq. (26) [2], which takes into account the traction force on the wheel and the air resistance. The dynamic coefficient is essentially the reserve of the traction force on the wheel reduced by the weight of the vehicle.

$$F_{rez,i}(v) = F_{K,i}(v) - R_{cel}(v) \tag{25}$$

where:

- $F_{rez,i}(v)$...... reserve of the traction force in i[th] gear

$$D_i(v) = \frac{F_{K,i}(v) - R_z(v)}{G} = \frac{R_f + R_j + R_i}{G} \tag{26}$$

where:

- $D_i(v)$ .... [/] dynamic coefficient of the vehicle
- G .... [N] weight of the vehicle

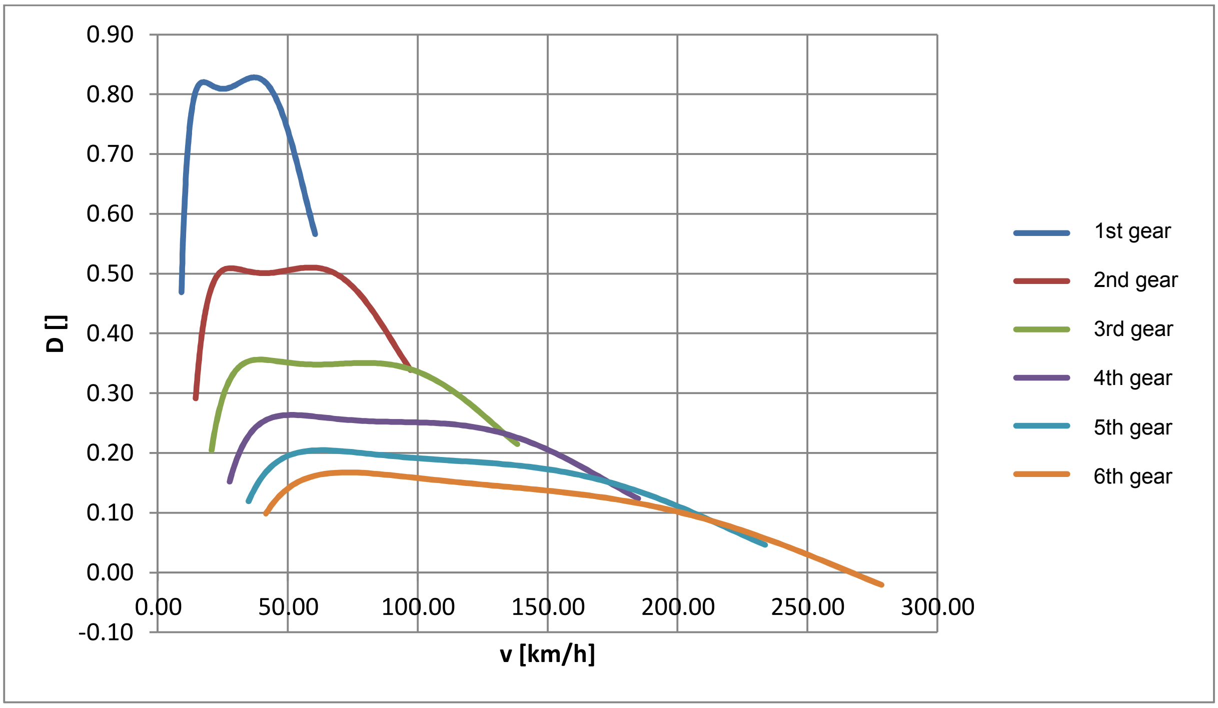


Figure 11: Diagram of the vehicle dynamic coefficient

Figure 11 shows the values of the dynamic coefficient depending on the vehicle (data: vehicle VW Golf GTI).

2.6.5 Limiting slopes

A relatively high driving resistance also represents the force required to overcome the slope. The power to overcome this force is greatly increased with the speed and becomes the main limitation of the maximum vehicle speed.
The maximum climb that the vehicle can handle at a certain speed is derived from Eq. (27) [2].

$$D(v) = f\cos(\alpha(v)) + \sin(\alpha(v)) = f\sqrt{1-\sin^2(\alpha(v))} + \sin(\alpha(v)) \tag{27}$$

It is assumed that the speed during the driving through the slope is constant. By squaring the equation we obtain

$$(1+f^2)\sin^2(\alpha(v)) - 2D(v)\sin(\alpha(v)) + (D^2(v) - f^2) = 0 \tag{28}$$

From there follows

$$\sin(\alpha(v)) = \frac{D(v) - f\sqrt{1+f^2-D^2(v)}}{1+f^2} \Rightarrow \tag{29}$$

$$\alpha_i(v) = \arcsin\left(\frac{D_i(v) - f\sqrt{1 - D_i^2(v) + f^2}}{1+f^2}\right) \tag{30}$$

The condition must be fulfilled 1 > *D(v)* > *f* in order for the squareroot to be real number.
The limiting slope represents the slope angle which the vehicle can still overcome at a certain constant speed.

- $\alpha_i(v)$......... [°] limiting slope

The size of the slope in practice is given in percentages (*j* = [%]) and not in degrees (α = [°]). The mathematical link between the two methods of giving a slope is given by the Eq. (31).

$$j = \tan(\alpha[^\circ])100[\%] \tag{31}$$

- *j* .... [%] slope intensity

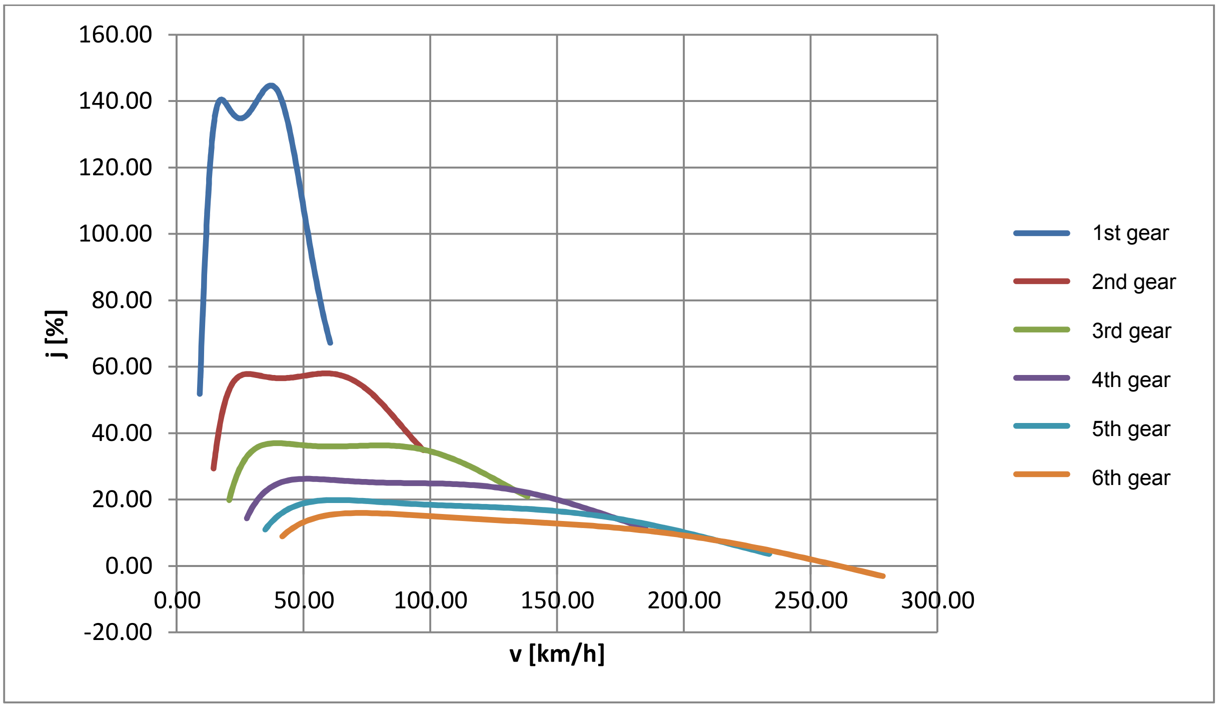


Figure 12: Diagram of limiting slopes

Figure 12 shows the limiting slopes depending on the vehicle speed for each gear (data: vehicle VW Golf GTI). It is obvious that the vehicle in 1st gear can overcome the slope of 140 %. This means that the vehicle has engine powerful enough to overcome that kind of slope but in practice this would not be possible since the tires would sleep or the vehicle would tip over the rear wheels.

2.6.6 Available power on wheels and power reserves (Power balance)

Instead of the balance of forces, we can carry out the balance of power inputs on the drive wheels P_K.

$$P_K = P_M \cdot \eta = P_f + P_s + P_z + P_i \tag{32}$$

The power brought to the driving wheels is equal to the total power required to overcome the driving resistance. This term is called the power balance. The power balance is very suitable for solving fuel economy problems as well as for analysing individual parameters of the motor vehicle system. Links between forces and power

$$\begin{aligned} P_f &= R_f \cdot v \quad [\mathrm{W}] \quad ; \quad P_s = R_s \cdot v \quad [\mathrm{W}] \\ P_z &= R_z \cdot v \quad [\mathrm{W}] \quad ; \quad P_i = R_i \cdot v \quad [\mathrm{W}] \end{aligned} \tag{33}$$

Curves 1-5 in Figure 13 represent the power on the wheels in a certain gear, while the thicker curve represents the sum of resistance powers at a certain speed. The intersection of this curve with the power curves determines the maximum speed of the vehicle.

In case of a numerical calculation of the characteristics, it is best to calculate the power balance with the following equations:

$$P_{i,(cel)}(v)=\frac{F_{i,(R_{cel})}(v)v}{3{,}6} \tag{34}$$

where:

- $P_{i,cel}(v)$ .... [kW]power on wheels depending on vehicle speed in ith gear
- $F_{i,(R_{cel})}(v)$. [N] traction force on the wheels in ith gear

Difference between the power on wheels and the total power of resistances *ΔP or* P_{rez} represents the power reserve which can be used for vehicle acceleration (see Figure 13).

$$P_{rez}=P_{i,(cel)}-P_{Rcel} \tag{35}$$

- P_{rez} .... [N] power reserve
- P_{Rcel}......... [N] total power of resistances

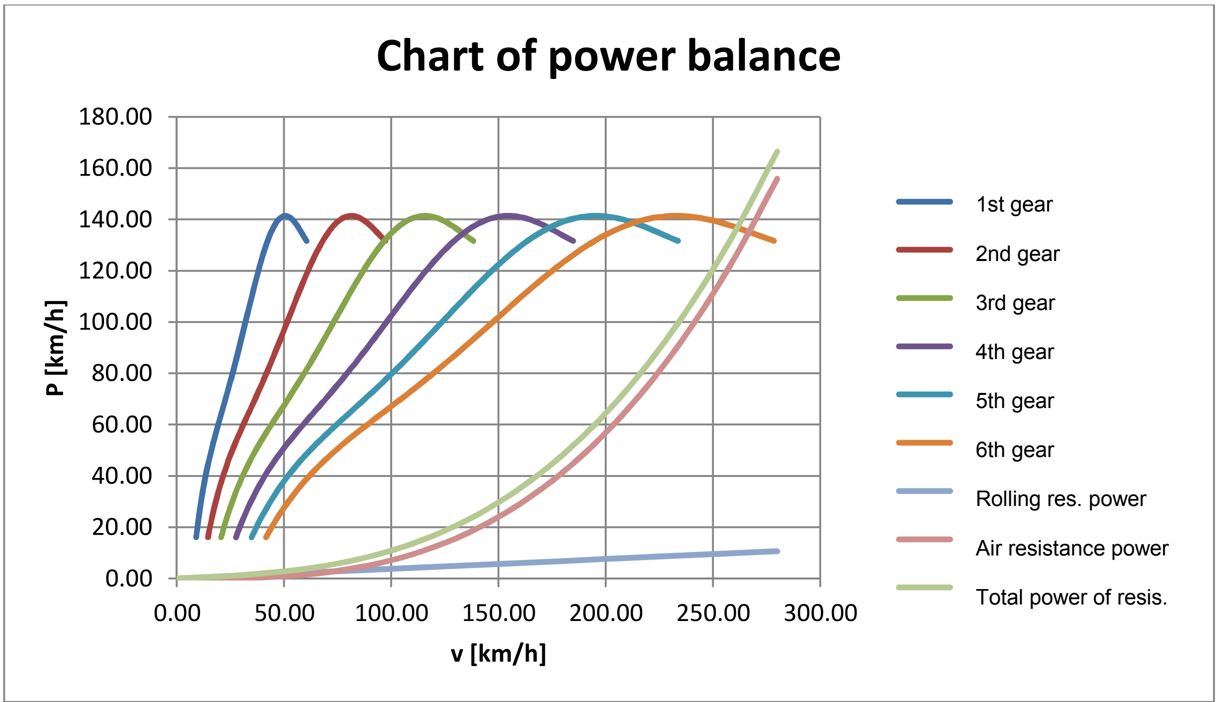


Figure 13: Power chart (data: vehicle VW Golf GTI)

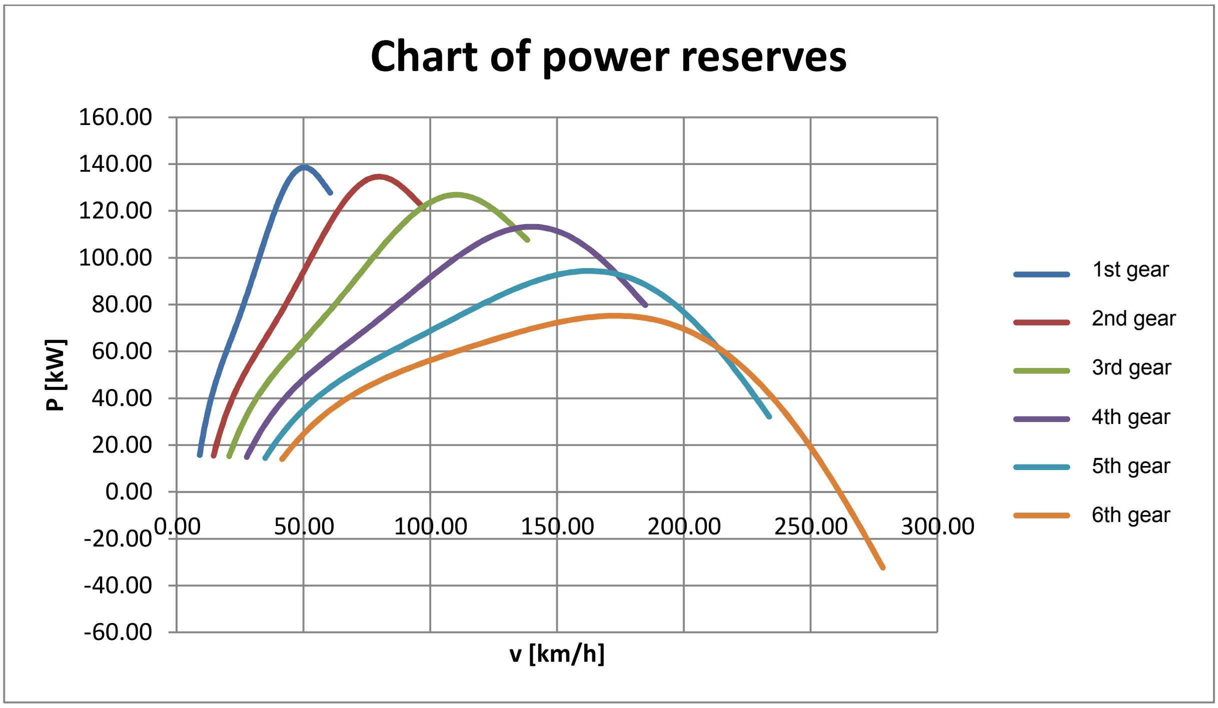


Figure 14: Diagram of power reserves (data: vehicle VW Golf GTI)

2.6.7 Vehicle acceleration

The most important result of the vehicle dynamics calculation is the acceleration of the vehicle, which we usually want to be as large as possible. The acceleration is calculated using the equation on the basis of already calculated values (36) [2].

$$a_i(v) = \left(D_i(v) - f\right)\frac{g}{\delta} \qquad (36)$$

where:

- $a_i(v)$ [m/s^2]vehicle acceleration
- g [m/s^2] .. gravity acceleration
- δ [/] .... coefficient of rotational masses

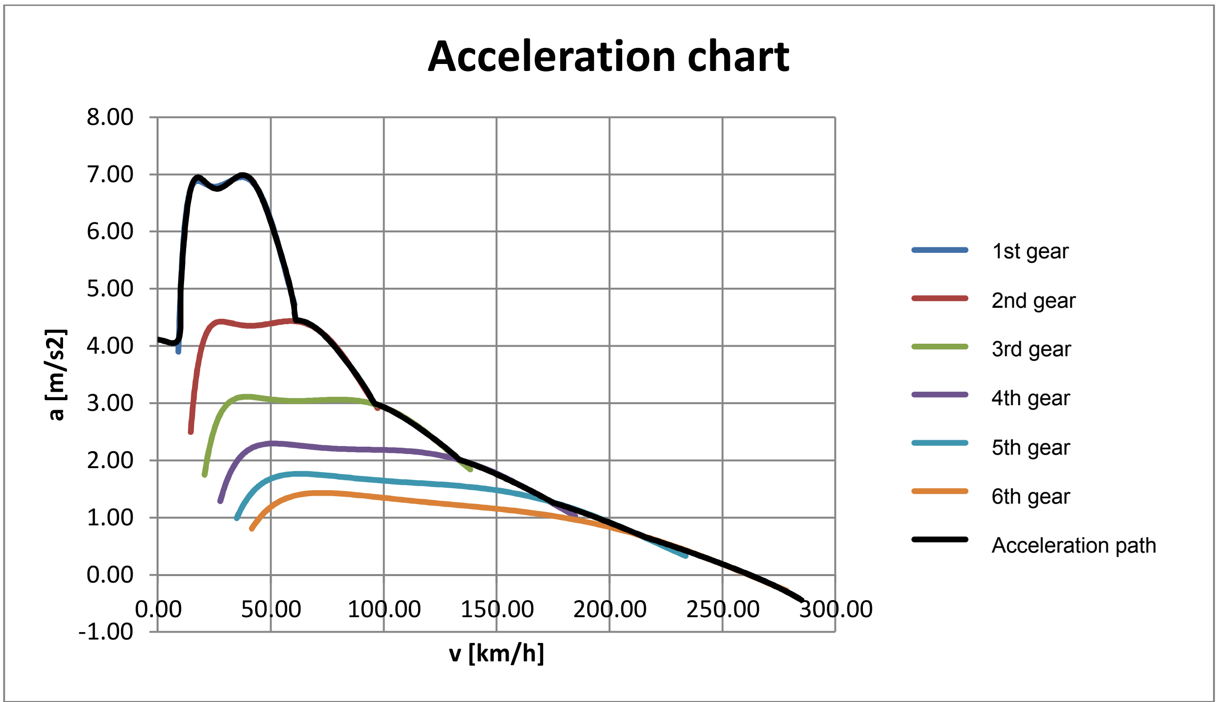


Figure 15: Diagram of the vehicle acceleration

Figure 15 shows the value of acceleration in different gears depending on the vehicle speed. The curve »Acceleration path« shows maximum possible acceleration from the start to the maximum vehicle speed (data: vehicle VW Golf GTI).

2.6.8 Acceleration time

When calculating the acceleration time, we proceed from the expression (37) from which we obtain the expression (38) for calculating the acceleration time between two speeds (v_1 and v_2)

$$a = \frac{dv}{dt} \Rightarrow dt = \frac{dv}{a} \tag{37}$$

$$t(v) = \int_{v_1}^{v_2} \frac{1}{a(v)} dv \tag{38}$$

where:

- $t(v)$ ...... [s] acceleration time from v_1 to v_2
- v_1 .... [m/s] vehicle start speed
- v_2 .... [m/s] vehicle end speed

During acceleration it is necessary to shift from lower gear to a higher one, which also requires a certain amount of time, so the actual accelerating times are greater than the times so calculated.

2.6.9 Path taken during the acceleration

When calculating the path taken during the acceleration, we proceed from the expression (39) from which we obtain the expression (40) for calculating the path taken during the acceleration between two speeds (v_1 and v_2).

$$v = \frac{dS}{dt} \Rightarrow dS = vdt \tag{39}$$

$$s(v) = \int_{t(v_1)}^{t(v_2)} v(t)dt \tag{40}$$

where

- *s(v)*.......... [m] path taken during the acceleration from v_1 to v_2
- $t(v_1)$ .... [s] time at vehicle start speed
- $t(v_2)$ .... [s] time at vehicle end speed

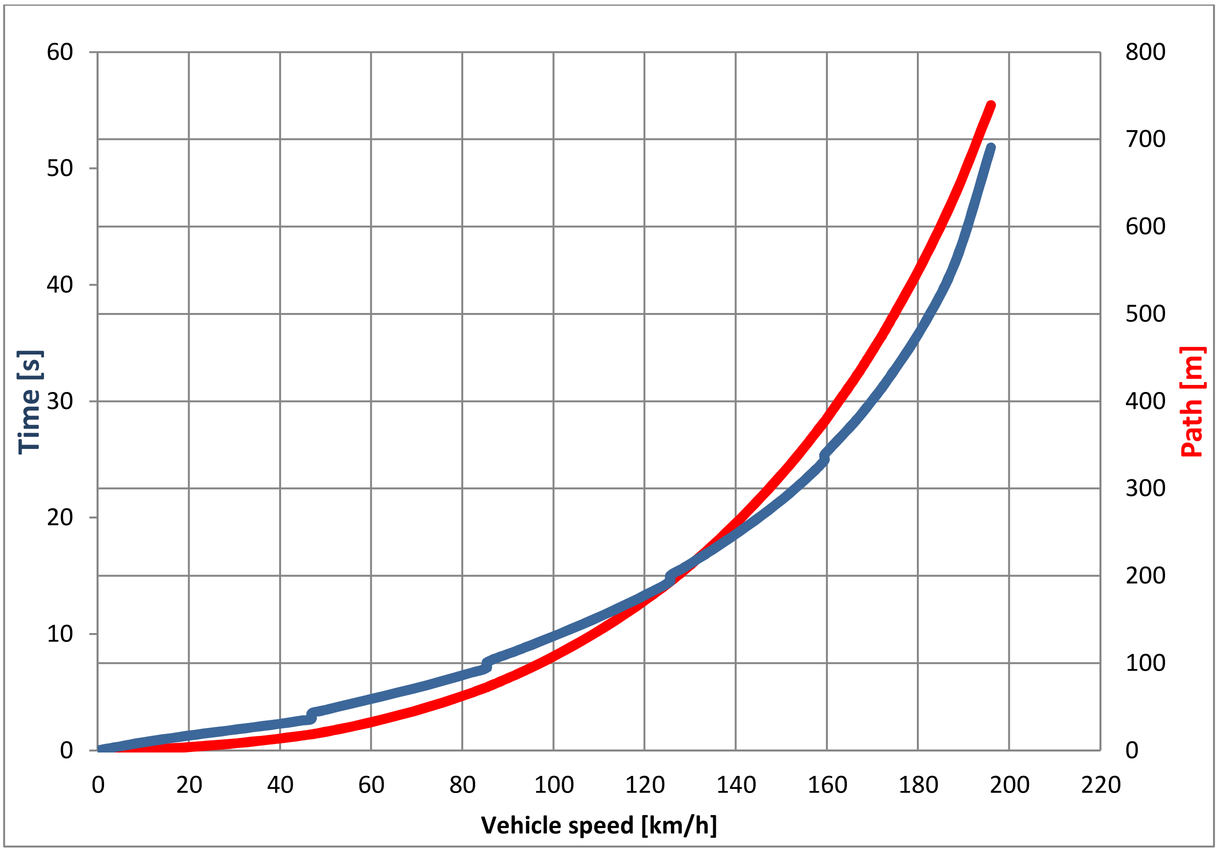


Figure 16: Diagram of acceleration time and path

Figure 16 shows time needed to accelerate to a certain speed path taken during the acceleration. This diagram also takes into account the time needed for gear shifting. Shifting time is t_{pres} = 0,5 s.

2.7 Instructions for numerical integration by Simpson

Simpson's rule

$$\int_a^b f(x)dx = \frac{h}{3}\left(f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right) + R \tag{41}$$

$$h = \frac{b-a}{2} \tag{42}$$

Generalized Simpson's rule

$$\int_a^b f(x)dx = \frac{h}{3}\left(f_0 + 4f_1 + 2f_2 + \ldots + 2f_{n-2} + 4f_{n-1} + f_n\right) \tag{43}$$

$$h = \frac{b-a}{n} \tag{44}$$

3. References

[1] Kraut, B.: Krautov strojniški priročnik, Tehniška založba Slovenije, Ljubljana, 1993

[2] Simić, D.: Motorna vozila, Naučna knjiga, Beograd, 1988