Designation: E 739 – 91 (Reapproved 2004)$^{\epsilon 1}$

Standard Practice for Statistical Analysis of Linear or Linearized Stress-Life (*S-N*) and Strain-Life (ϵ-*N*) Fatigue Data[1]

This standard is issued under the fixed designation E 739; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

ϵ^1 NOTE—Editorial changes were made throughout in May 2006.

1. Scope

1.1 This practice covers only *S-N* and ϵ-*N* relationships that may be reasonably approximated by a straight line (on appropriate coordinates) for a specific interval of stress or strain. It presents elementary procedures that presently reflect good practice in modeling and analysis. However, because the actual *S-N* or ϵ-*N* relationship is approximated by a straight line only within a specific interval of stress or strain, and because the actual fatigue life distribution is unknown, it is *not recommended* that (*a*) the *S-N* or ϵ-*N* curve be extrapolated outside the interval of testing, or (*b*) the fatigue life at a specific stress or strain amplitude be estimated below approximately the fifth percentile ($P \simeq 0.05$). As alternative fatigue models and statistical analyses are continually being developed, later revisions of this practice may subsequently present analyses that permit more complete interpretation of *S-N* and ϵ-*N* data.

2. Referenced Documents

2.1 *ASTM Standards:* [2]

E 206 Definitions of Terms Relating to Fatigue Testing and the Statistical Analysis of Fatigue Data[3]

E 467 Practice for Verification of Constant Amplitude Dynamic Forces in an Axial Fatigue Testing System

E 468 Practice for Presentation of Constant Amplitude Fatigue Test Results for Metallic Materials

E 513 Definitions of Terms Relating to Constant-Amplitude, Low-Cycle Fatigue Testing[3]

E 606 Practice for Strain-Controlled Fatigue Testing

3. Terminology

3.1 The terms used in this practice shall be used as defined in Definitions E 206 and E 513. In addition, the following terminology is used:

3.1.1 *dependent variable*—the fatigue life *N* (or the logarithm of the fatigue life).

3.1.1.1 *Discussion*—Log (*N*) is denoted *Y* in this practice.

3.1.2 *independent variable*—the selected and controlled variable (namely, stress or strain). It is denoted *X* in this practice when plotted on appropriate coordinates.

3.1.3 *log-normal distribution*—the distribution of *N* when log (*N*) is normally distributed. (Accordingly, it is convenient to analyze log (*N*) using methods based on the normal distribution.)

3.1.4 *replicate (repeat) tests*—nominally identical tests on different randomly selected test specimens conducted at the same nominal value of the independent variable *X*. Such replicate or repeat tests should be conducted independently; for example, each replicate test should involve a separate set of the test machine and its settings.

3.1.5 *run out*—no failure at a specified number of load cycles (Practice E 468).

3.1.5.1 *Discussion*—The analyses illustrated in this practice do not apply when the data include either run-outs (or suspended tests). Moreover, the straight-line approximation of the *S-N* or ϵ-*N* relationship may not be appropriate at long lives when run-outs are likely.

3.1.5.2 *Discussion*—For purposes of statistical analysis, a run-out may be viewed as a test specimen that has either been removed from the test or is still running at the time of the data analysis.

4. Significance and Use

4.1 Materials scientists and engineers are making increased use of statistical analyses in interpreting *S-N* and ϵ-*N* fatigue data. Statistical analysis applies when the given data can be reasonably assumed to be a random sample of (or representation of) some specific defined population or universe of

[1] This practice is under the jurisdiction of ASTM Committee E08 on Fatigue and Fracture and is the direct responsibility of Subcommittee E08.04 on Structural Applications.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

[3] Withdrawn.

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material of interest (under specific test conditions), and it is desired either to characterize the material or to predict the performance of future random samples of the material (under similar test conditions), or both.

5. Types of *S-N* and ϵ-*N* Curves Considered

5.1 It is well known that the shape of *S-N* and ϵ-*N* curves can depend markedly on the material and test conditions. This practice is restricted to linear or linearized *S-N* and ϵ-*N* relationships, for example,

$$\log N = A + B(S) \text{ or} \tag{1}$$

$$\log N = A + B(\epsilon) \text{ or}$$

$$\log N = A + B(\log S) \text{ or} \tag{2}$$

$$\log N = A + B(\log \epsilon)$$

in which S and ϵ may refer to (*a*) the maximum value of constant-amplitude cyclic stress or strain, given a specific value of the stress or strain ratio, or of the minimum cyclic stress or strain, (*b*) the amplitude or the range of the constant-amplitude cyclic stress or strain, given a specific value of the mean stress or strain, or (*c*) analogous information stated in terms of some appropriate independent (controlled) variable.

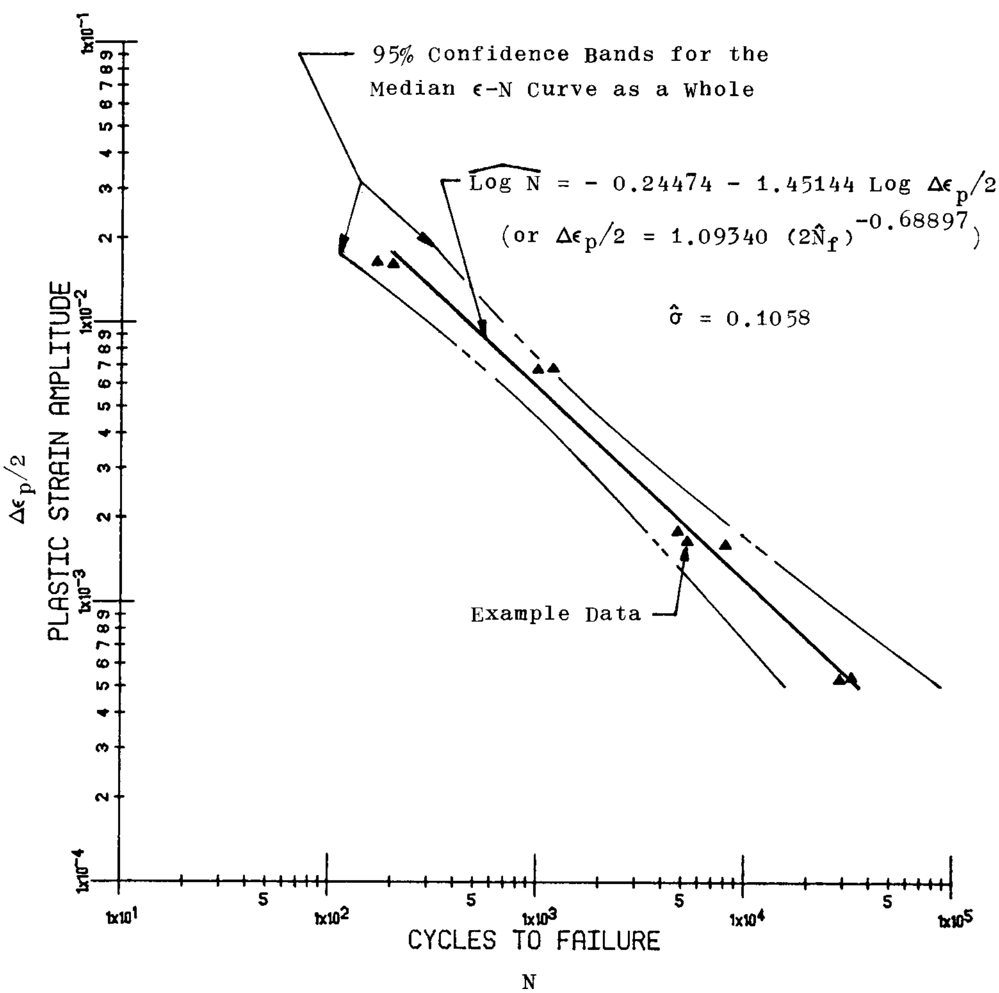
NOTE 1—In certain cases, the amplitude of the stress or strain is not constant during the entire test for a given specimen. In such cases some effective (equivalent) value of S or ϵ must be established for use in analysis.

5.1.1 The fatigue life N is the dependent (random) variable in *S-N* and ϵ-*N* tests, whereas S or ϵ is the independent (controlled) variable.

NOTE 2—In certain cases, the independent variable used in analysis is not literally the variable controlled during testing. For example, it is common practice to analyze low-cycle fatigue data treating the range of plastic strain as the controlled variable, when in fact the range of total strain was actually controlled during testing. Although there may be some question regarding the exact nature of the controlled variable in certain *S-N* and ϵ-*N* tests, there is never any doubt that the fatigue life is the dependent variable.

NOTE 3—In plotting *S-N* and ϵ-*N* curves, the independent variables S and ϵ are plotted along the ordinate, with life (the dependent variable) plotted along the abscissa. Refer, for example, to Fig. 1.

5.1.2 The distribution of fatigue life (in any test) is unknown (and indeed may be quite complex in certain situations). For



NOTE 1—The 95 % confidence band for the ϵ-*N* curve as a whole is based on Eq 10. (Note that the dependent variable, fatigue life, is plotted here along the abscissa to conform to engineering convention.)

FIG. 1 Fitted Relationship Between the Fatigue Life N (Y) and the Plastic Strain Amplitude $\Delta\epsilon_p/2$ (X) for the Example Data Given

the purposes of simplifying the analysis (while maintaining sound statistical procedures), it is assumed in this practice that the logarithms of the fatigue lives are normally distributed, that is, the fatigue life is log-normally distributed, and that the variance of log life is constant over the entire range of the independent variable used in testing (that is, the scatter in log N is assumed to be the same at low S and ϵ levels as at high levels of S or ϵ). Accordingly, log N is used as the dependent (random) variable in analysis. It is denoted Y. The independent variable is denoted X. It may be either S or ϵ, or log S or log ϵ, respectively, depending on which appears to produce a straight line plot for the interval of S or ϵ of interest. Thus Eq 1 and Eq 2 may be re-expressed as

$$Y = A + BX \tag{3}$$

Eq 3 is used in subsequent analysis. It may be stated more precisely as $\mu_{Y|X} = A + BX$, where $\mu_{Y|X}$ is the expected value of Y given X.

NOTE 4—For testing the adequacy of the linear model, see 8.2.

NOTE 5—The expected value is the mean of the conceptual population of all Y's given a specific level of X. (The median and mean are identical for the symmetrical normal distribution assumed in this practice for Y.)

6. Test Planning

6.1 Test planning for S-N and ϵ-N test programs is discussed in Chapter 3 of Ref (**1**).[4] Planned grouping (blocking) and randomization are essential features of a well-planned test program. In particular, good test methodology involves use of planned grouping to (*a*) balance potentially spurious effects of nuisance variables (for example, laboratory humidity) and (*b*) allow for possible test equipment malfunction during the test program.

7. Sampling

7.1 It is vital that sampling procedures be adopted that assure a random sample of the material being tested. A random sample is required to state that the test specimens are representative of the conceptual universe about which both statistical and engineering inference will be made.

NOTE 6—A random sampling procedure provides each specimen that conceivably could be selected (tested) an equal (or known) opportunity of actually being selected at each stage of the sampling process. Thus, it is poor practice to use specimens from a single source (plate, heat, supplier) when seeking a random sample of the material being tested unless that particular source is of specific interest.

NOTE 7—Procedures for using random numbers to obtain random samples and to assign stress or strain amplitudes to specimens (and to establish the time order of testing) are given in Chapter 4 of Ref (**2**).

7.1.1 *Sample Size*—The minimum number of specimens required in S-N (and ϵ-N) testing depends on the type of test program conducted. The following guidelines given in Chapter 3 of Ref (**1**) appear reasonable.

Type of Test	Minimum Number of Specimens[A]
Preliminary and exploratory (exploratory research and development tests)	6 to 12
Research and development testing of components and specimens	6 to 12
Design allowables data	12 to 24
Reliability data	12 to 24

[A] If the variability is large, a wide confidence band will be obtained unless a large number of specimens are tested (See 8.1.1).

7.1.2 *Replication*—The replication guidelines given in Chapter 3 of Ref (**1**) are based on the following definition:

% replication = 100 [1 – (total number of different stress or strain levels used in testing/total number of specimens tested)]

Type of Test	Percent Replication[A]
Preliminary and exploratory (research and development tests)	17 to 33 min
Research and development testing of components and specimens	33 to 50 min
Design allowables data	50 to 75 min
Reliability data	75 to 88 min

[A] Note that percent replication indicates the portion of the total number of specimens tested that may be used for obtaining an estimate of the variability of replicate tests.

7.1.2.1 *Replication Examples*—Good replication: Suppose that ten specimens are used in research and development for the testing of a component. If two specimens are tested at each of five stress or strain amplitudes, the test program involves 50 % replications. This percent replication is considered adequate for most research and development applications. Poor replication: Suppose eight different stress or strain amplitudes are used in testing, with two replicates at each of two stress or strain amplitudes (and no replication at the other six stress or strain amplitudes). This test program involves only 20 % replication, which is not generally considered adequate.

8. Statistical Analysis (Linear Model $Y = A + BX$, Log-Normal Fatigue Life Distribution with Constant Variance Along the Entire Interval of X Used in Testing, No Runouts or Suspended Tests or Both, Completely Randomized Design Test Program)

8.1 For the case where (*a*) the fatigue life data pertain to a random sample (all Y_i are independent), (*b*) there are neither run-outs nor suspended tests and where, for the entire interval of X used in testing, (*c*) the S-N or ϵ-N relationship is described by the linear model $Y = A + BX$ (more precisely by $\mu_{Y|X} = A + BX$), (*d*) the (two parameter) log-normal distribution describes the fatigue life N, and (*e*) the variance of the log-normal distribution is constant, the maximum likelihood estimators of A and B are as follows:

$$\hat{A} = \bar{Y} - \hat{B}\bar{X} \tag{4}$$

$$\hat{B} = \frac{\sum_{i=1}^{k} (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{k} (X_i - \bar{X})^2} \tag{5}$$

where the symbol "caret" (^) denotes estimate (estimator), the symbol "overbar"(¯) denotes average (for example, $\bar{Y} = \sum_{i=1}^{k} Y_i/k$ and $\bar{X} = \sum_{i=1}^{k} X_i/k$), $Y_i = \log N_i$, $X_i = S_i$ or ϵ_i, or log S_i or log ϵ_i (refer to Eq 1 and Eq 2), and k is the total number of test specimens (the total sample size). The recommended expression for estimating the variance of the normal distribution for log N is

[4] The boldface numbers in parentheses refer to the list of references appended to this standard.

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{k} (Y_i - \hat{Y}_i)^2}{k-2} \tag{6}$$

in which $\hat{Y}_i = \hat{A} + \hat{B}X_i$ and the $(k - 2)$ term in the denominator is used instead of k to make $\hat{\sigma}^2$ an unbiased estimator of the normal population variance σ^2.

NOTE 8—An assumption of constant variance is usually reasonable for notched and joint specimens up to about 10^6 cycles to failure. The variance of unnotched specimens generally increases with decreasing stress (strain) level (see Section 9). If the assumption of constant variance appears to be dubious, the reader is referred to Ref (**3**) for the appropriate statistical test.

8.1.1 *Confidence Intervals for Parameters A and B*—The estimators $\hat{A}$ and $\hat{B}$ are normally distributed with expected values A and B, respectively, (regardless of total sample size k) when conditions (*a*) through (*e*) in 8.1 are met. Accordingly, confidence intervals for parameters A and B can be established using the t distribution, Table 1. The confidence interval for A is given by $\hat{A} \pm t_p \hat{\sigma}_{\hat{A}}$, or

$$\hat{A} \pm t_p\, \hat{\sigma} \left[\frac{1}{k} + \frac{\bar{X}^2}{\sum_{i=1}^{k} (X_i - \bar{X})^2} \right]^{1/2}, \tag{7}$$

and for B is given by $\hat{B} \pm t_p \hat{\sigma}_{\hat{B}}$, or

$$\hat{B} \pm t_p \hat{\sigma} [\sum_{i=1}^{k} (X_i - \bar{X})^2]^{-1/2} \tag{8}$$

in which the value of t_p is read from Table 1 for the desired value of P, the confidence level associated with the confidence interval. This table has one entry parameter (the statistical degrees of freedom, n, for t). For Eq 7 and Eq 8, $n = k - 2$.

NOTE 9—The confidence intervals for A and B are exact if conditions (*a*) through (*e*) in 8.1 are met exactly. However, these intervals are still reasonably accurate when the actual life distribution differs slightly from the (two-parameter) log-normal distribution, that is, when only condition (*d*) is not met exactly, due to the robustness of the t statistic.

NOTE 10—Because the actual median *S-N* or ϵ-*N* relationship is only approximated by a straight line within a specific interval of stress or strain, confidence intervals for A and B that pertain to confidence levels greater than approximately 0.95 are not recommended.

TABLE 1 Values of t_p (Abstracted from STP 313 (4))

n^A	P, %^{B}	
	90	95
4	2.1318	2.7764
5	2.0150	2.5706
6	1.9432	2.4469
7	1.8946	2.3646
8	1.8595	2.3060
9	1.8331	2.2622
10	1.8125	2.2281
11	1.7959	2.2010
12	1.7823	2.1788
13	1.7709	2.1604
14	1.7613	2.1448
15	1.7530	2.1315
16	1.7459	2.1199
17	1.7396	2.1098
18	1.7341	2.1009
19	1.7291	2.0930
20	1.7247	2.0860
21	1.7207	2.0796
22	1.7171	2.0739

^{A} n is not sample size, but the degrees of freedom of t, that is, $n = k - 2$.
^{B} P is the probability in percent that the random variable t lies in the interval from $-t_p$ to $+t_p$.

8.1.1.1 The meaning of the confidence interval associated with, say, Eq 8 is as follows (Note 11). If the values of t_p given in Table 1 for, say, $P = 95$ % are used in a series of analyses involving the estimation of B from independent data sets, then in the long run we may expect 95 % of the computed intervals to include the value B. If in each instance we were to assert that B lies within the interval computed, we should expect to be correct 95 times in 100 and in error 5 times in 100: that is, the statement "B lies within the computed interval" has a 95 % probability of being correct. But there would be no operational meaning in the following statement made in any one instance: "The probability is 95 % that B falls within the computed interval in this case" since B either does or does not fall within the interval. It should also be emphasized that even in independent samples from the same universe, the intervals given by Eq 8 will vary both in width and position from sample to sample. (This variation will be particularly noticeable for small samples.) It is this series of (random) intervals "fluctuating" in size and position that will include, ideally, the value B 95 times out of 100 for $P = 95$ %. Similar interpretations hold for confidence intervals associated with other confidence levels. For a given total sample size k, it is evident that the width of the confidence interval for B will be a minimum whenever

$$\sum_{i=1}^{k} (X_i - \bar{X})^2 \tag{9}$$

is a maximum. Since the X_i levels are selected by the investigator, the width of confidence interval for B may be reduced by appropriate test planning. For example, the width of the interval will be minimized when, for a fixed number of available test specimens, k, half are tested at each of the extreme levels $X_{\min}$ and $X_{\max}$. However, this allocation should be used only when there is strong *a priori* knowledge that the *S-N* or ϵ-*N* curve is indeed linear—because this allocation precludes a statistical test for linearity (8.2). See Chapter 3 of Ref (**1**) for a further discussion of efficient selection of stress (or strain) levels and the related specimen allocations to these stress (or strain) levels.

NOTE 11—This explanation is similar to that of STP 313 (**4**).

8.1.2 *Confidence Band for the Entire Median S-N or ϵ-N Curve (that is, for the Median S-N or ϵ-N Curve as a Whole)*—If conditions (*a*) through (*e*) in 8.1 are met, an exact confidence band for the entire median *S-N* or ϵ-*N* curve (that is, all points on the linear or linearized median *S-N* or ϵ-*N* curve considered simultaneously) may be computed using the following equation:

$$\hat{A} + \hat{B}X \pm \sqrt{2F_p}\hat{\sigma} \left[\frac{1}{k} + \frac{(X - \bar{X})^2}{\sum_{i=1}^{k} (X_i - \bar{X})^2} \right]^{1/2} \tag{10}$$

in which F_p is given in Table 2. This table involves two entry parameters (the statistical degrees of freedom n_1 and n_2 for F). For Eq 9, $n_1 = 2$ and $n_2 = (k - 2)$. For example, when $k = 7$, $F_{0.95} = 5.7861$.

TABLE 2 Values of F_P[A] (Abstracted from STP 313 (4))

		Degrees of Freedom, n_1			
		1	2	3	4
	1 {	161.45	199.50	215.71	224.58
		4052.2	4999.5	5403.3	5624.6
	2 {	18.513	19.000	19.164	19.247
		8.503	99.000	99.166	99.249
	3 {	10.128	9.5521	9.2766	9.1172
		34.116	30.817	29.457	28.710
	4 {	7.7086	6.9443	6.5914	6.3883
		21.198	18.000	16.694	15.977
	5 {	6.6079	5.7861	5.4095	5.1922
		16.258	13.274	12.060	11.392
	6 {	5.9874	5.1433	4.7571	4.5337
		13.745	10.925	9.7795	9.1483
	7 {	5.5914	4.7374	4.3468	4.1203
		12.246	9.5466	8.4513	7.8467
	8 {	5.3177	4.4590	4.0662	3.8378
Degrees of Freedom, n_2		11.259	8.6491	7.5910	7.0060
	9 {	5.1174	4.2565	3.8626	3.6331
		10.561	8.0215	6.9919	6.4221
	10 {	4.9646	4.1028	3.7083	3.4780
		10.044	7.5594	6.5523	5.9943
	11 {	4.8443	3.9823	3.5874	3.3567
		9.6460	7.2057	6.2167	5.6683
	12 {	4.7472	3.8853	3.4903	3.2592
		9.3302	6.9266	5.9526	5.4119
	13 {	4.6672	3.8056	3.4105	3.1791
		9.0738	6.7010	5.7394	5.2053
	14 {	4.6001	3.7389	3.3439	3.1122
		8.8616	6.5149	5.5639	5.0354
		4.5431	3.6823	3.2874	3.0556
	15 {	8.6831	6.3589	5.4170	4.8932

[A] In each row, the top figures are values of F corresponding to P = 95 %, the bottom figures correspond to P = 99 %. Thus, the top figures pertain to the 5 % significance level, whereas the bottom figures pertain to the 1 % significance level. (The bottom figures are not recommended for use in Eq 10.)

8.1.2.1 A 95 % confidence band computed using Eq 10 is plotted in Fig. 1 for the example data of 8.3.1. The interpretation of this band is similar to that for a confidence interval (8.1.1). Namely, if conditions (*a*) through (*e*) are met, and if the values of F_p given in Table 2 for, say, P = 95 % are used in a series of analyses involving the construction of confidence bands using Eq 10 for the entire range of X used in testing; then in the long run we may expect 95 % of the computed hyperbolic bands to include the straight line $\mu_{Y|X} = A + BX$ everywhere along the entire range of X used in testing.

NOTE 12—Because the actual median *S-N* or ϵ-*N* relationship is only approximated by a straight line within a specific interval of stress of strain, confidence bands which pertain to confidence levels greater than approximately 0.95 are not recommended.

8.1.2.2 While the hyperbolic confidence bands generated by Eq 9 and plotted in Fig. 1 are statistically correct, straight-line confidence and tolerance bands parallel to the fitted line $\hat{\mu}_{Y|X} = \hat{A} + B$ are sometimes used. These bands are described in Chapter 5 of Ref **(2)**.

8.2 *Testing the Adequacy of the Linear Model*—In 8.1, it was assumed that a linear model is valid, namely that $\mu_{Y|X} = A + BX$. If the test program is planned such that there is more than one observed value of Y at some of the X_i levels where $i \geq 3$, then a statistical test for linearity can be made based on the F distribution, Table 2. The log life of the jth replicate specimen tested in the ith level of X is subsequently denoted Y_{ij}.

8.2.1 Suppose that fatigue tests are conducted at l different levels of X and that m_i replicate values of Y are observed at each X_i. Then the hypothesis of linearity (that $\mu_{Y|X} = A + BX$) is rejected when the computed value of

$$\frac{\sum_{i=1}^{l} m_i\,(\hat{Y}_i - \bar{Y}_i)^2/(l-2)}{\sum_{i=1}^{l}\sum_{j=1}^{m_i}(Y_{ij} - \bar{Y}_i)^2/(k-l)} \qquad (11)$$

exceeds F_p, where the value of F_p is read from Table 2 for the desired significance level. (The significance level is defined as the probability in percent of incorrectly rejecting the hypothesis of linearity when there is indeed a linear relationship between X and $\mu_{Y|X}$.) The total number of specimens tested, k, is computed using

$$k = \sum_{i=1}^{l} m_i \qquad (12)$$

8.2.2 Table 2 involves two entry parameters (the statistical degrees of freedom n_1 and n_2 for F). For Eq 11, $n_1 = (l-2)$, and $n_2 = (k-l)$. For example, $F_{0.95} = 6.9443$ when $k = 8$ and $l = 4$.

8.2.3 The F test (Eq 11) compares the variability of average value about the fitted straight line, as measured by their mean square (Note 14) (the numerator in Eq 11) to the variability among replicates, as measured by their mean square (the denominator in Eq 11). The latter mean square is independent of the form of the model assumed for the *S-N* or ϵ-*N* relationship. If the relationship between $\mu_{Y|X}$ and X is indeed linear, Eq 11 follows the F distribution with degrees of freedom, $(l-2)$ and $(k-l)$. Otherwise Eq 11 is larger on the average than would be expected by random sampling from this

F distribution. Thus the hypothesis of a linear model is rejected if the observed value of F (Eq 11) exceeds the tabulated value F_p. If the linear model is rejected, it is recommended that a nonlinear model be considered, for example:

$$\mu_{Y|X} A + BX + CX^2 \tag{13}$$

NOTE 13—Some readers may be tempted to use existing digital computer software which calculates a value of r, the so-called correlation coefficient, or r^2, the coefficient of determination, to ascertain the suitability of the linear model. This approach is not recommended. (For example, $r = 0.993$ with $F = 3.62$ for the example of 8.3.1, whereas $r = 0.988$ and $F = 21.5$ for similar data set generated during the 1976 E09.08 low-cycle fatigue round robin.)

NOTE 14—A mean square value is a specific sum of squares divided by its statistical degrees of freedom.

8.3 *Numerical Examples*:

8.3.1 *Example 1:* Consider the following low-cycle fatigue data (taken from a 1976 E09.08 round-robin test program (laboratory 43):

$\Delta\epsilon_p/2$ Plastic Strain Amplitude—Unitless	N Fatigue Life Cycles
0.01636	168
0.01609	200
0.00675	1 000
0.00682	1 180
0.00179	4 730
0.00160	8 035
0.00165	5 254
0.00053	28 617
0.00054	32 650

8.3.1.1 Estimate parameters A and B and the respective 95 % confidence intervals.

8.3.1.2 First, restate (transform) the data in terms of logarithms (base 10 used in this practice due to its wide use in practice).

$X_i = \log (\Delta\epsilon_{pi}/2)$ (Independent Variable)	$Y_i = \log N_i$ (Dependent Variable)
−1.78622	2.22531
−1.79344	2.30103
−2.17070	3.00000
−2.16622	3.07188
−2.74715	3.67486
−2.79588	3.90499
−2.78252	3.72049
−3.27572	4.45662
−3.26761	4.51388

8.3.1.3 Then, from Eq 4 and Eq 5:

$\hat{A} = -0.24474$ $\hat{B} = -1.45144$

Or, as expressed in the form of Eq 2b:

$$\widehat{\log N} = -0.24474 - 1.45144 \log (\Delta\epsilon_p/2)$$

Also, from Eq 6:

$$\hat{\sigma}^2 = 0.07837/7 = 0.011195 \tag{14}$$

or,

$$\hat{\sigma} = 0.1058 \tag{15}$$

8.3.1.4 Accordingly, using Eq 7, the 95 % confidence interval for A is ($t_p = 2.3646$) [−0.6435, 0.1540], and, using Eq 8, the 95 % confidence interval for B is [−1.6054, − 1.2974].

8.3.1.5 The fitted line $\hat{Y} = \log N = -0.24474 - 1.45144 \log (\Delta\epsilon_p/2) = -0.24474 - 1.45144X$ is displayed in Fig. 1, where the 95 % confidence band computed using Eq 10 is also plotted. (For example, when $\Delta\epsilon_p/2 = 0.01$, $X = -2.000$, $\hat{Y} = 2.65814$, $\hat{Y}_{\text{lower band}} = 2.65814 - 0.15215 = 2.50599$, and $\hat{Y}_{\text{upper band}} = 2.65814 + 0.15215 = 2.81029$.)

8.3.1.6 The fitted line can be transformed to the form given in Appendix X1 of Practice E 606 as follows:

$$\log N = -0.24474 - 1.45144 \log (\Delta\epsilon_p/2) \tag{16}$$

$$\log (\Delta\epsilon_p/2) = -0.16862 - 0.68897 \log N$$

$$\Delta\epsilon_p/2 = 0.67823 \ (N)^{-0.68897}$$

Substituting cycles (N) to reversals ($2N_f$) gives

$$\Delta\epsilon_p/2 = 0.67823 \left(\frac{2\hat{N}_f}{2}\right)^{-0.68897} \tag{17}$$

$$\Delta\epsilon_p/2 = 0.67823\,(1/2)^{-0.68897}\,(2N_f)^{-0.68897}$$

$$\Delta\epsilon_p/2 = 1.09340\,(2\hat{N}_f)^{-0.68897}$$

The above alternative equation is shown on Fig. 1.

8.3.1.7 *Ancillary Calculations:*

$$\bar{X} = -2.53172 \qquad \bar{Y} = 3.42990 \tag{18}$$

$$\sum_{i=1}^{9} (X_i - X)^2 = 2.63892 \tag{19}$$

$$\sum_{i=1}^{9} (X_i - \bar{X})(Y_i - \bar{Y}) = -3.83023 \tag{20}$$

$$\hat{\sigma}_{\hat{A}} = \hat{\sigma}\left[\frac{1}{9} + \frac{(-2.53172)^2}{2.63892}\right]^{\frac{1}{2}} = 0.1686 \tag{21}$$

$$\hat{\sigma}_{\hat{B}} = \sigma[2.63892]^{-\frac{1}{2}} = 0.06513 \tag{22}$$

8.3.1.8 Test for linearity at the 5 % significance level.

8.3.1.9 We shall ignore the slight differences among the amplitudes of plastic strain and assume that $l = 4$ and $k = 9$. Then, at each of the four X_i levels, we shall compute $\hat{Y}_i$ using $\hat{Y}_i = -0.24414 - 1.45144\,\bar{X}_i$ and $\bar{Y}_i$ using $\bar{Y}_i = \Sigma Y_{ij}/m_i$. Accordingly, $F_{0.95} = 5.79$, whereas F computed (using Eq 11) = 3.62. Hence, we do not reject the linear model in this example.

8.3.1.10 *Ancillary Calculations:*

$$\text{Numerator } (F) = 0.0532/2 \tag{23}$$

$$\text{Denominator } (F) = 0.0368/5$$

8.3.2 *Example 2:* Consider the following low-cycle fatigue data (also taken from a 1976 E09.08 round-robin test program (laboratory 34)):

$\Delta\epsilon_p/2$ Plastic Strain Amplitude—Unitless	N Fatigue Life Cycles
0.0164	153
0.0164	153
0.0069	563
0.0069	694
0.00185	3 515
0.00175	3 860
0.00054	17 500
0.00058	20 330
0.000006	60 350
0.000006	121 500

8.3.2.1 The F test (Eq 11) in this case indicates that the linear model should be rejected at the 5 % significance level (that is, F calculated = 9.08, where $F_{3,5,0.95} = 5.41$). Hence estimation of *A* and *B* for the linear model is not recommended. Rather, a nonlinear model should be considered in analysis.

9. Other Statistical Analyses

9.1 When the Weibull distribution is assumed to describe the distribution of fatigue life at a given stress or strain amplitude, or when the fatigue data include either run-outs or suspended tests (or when the variance of log life increases noticeably as life increases), the appropriate statistical analyses are more complicated than illustrated in this practice. The reader is referred to Ref **(5)** for an example of relevant digital computer software.

NOTE 15—It is not good practice either to ignore run-outs or to treat them as if they were failures. Rather, maximum likelihood analyses of the type illustrated in Ref **(5)** are recommended.

REFERENCES

(1) *Manual on Statistical Planning and Analysis for Fatigue Experiments, STP 588*, ASTM International, 1975.

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(3) Brownlee, K. A., *Statistical Theory and Methodology in Science and Engineering*, John Wiley and Sons, New York, NY, 2nd Ed. 1965.

(4) *ASTM Manual on Fitting Straight Lines, STP 313*, ASTM International, 1962.

(5) Nelson, W. B., et al., "STATPAC Simplified—A Short Introduction To How To Run STATPAC, A General Statistical Package for Data Analysis," *Technical Information Series Report 73CRD 046*, July, 1973, General Electric Co., Corporate Research and Development, Schenectady, NY.

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